Karshi Multidisciplinary International Scientific Journal

www.kmisj.reapress.com

K. Multidiscip. Int. Sci. J. Vol. 2, No. 3 (2026) 124–151.

Paper Type: Original Article

A Review of Neutrosophic Sustainable Inventory Models

Anthriyas Theeba[1,2*], Florentin Smarandache[3], Nivetha Martin[4,5]

[1] Department of Mathematics, Arul Anandar College (Autonomous), Karumathur, Madurai, India; theebajoe@gmail.com.

[2] G.T.N. Arts College (Autonomous), Dindigul, Madurai Kamaraj University, Madurai, India.

[3] Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA; smarand@unm.edu.

[4] Department of Mathematics, Arul Anandar College, Karumathur, Madurai, Tamil Nadu, India; nivetha.martin710@gmail.com.

[5] Madurai Kamaraj University, Madurai, India.

Citation:

Received: 25 December 2024 *Revised: 19 February 2025* *Accepted: 17 April 2025*	Theeba, A., Smarandache, F., & Martin, N. (2025). A review of Neutrosophic sustainable inventory models. *Karshi multidisciplinary international scientific journal, 2* (3), 124-151.

Abstract

Sustainable Inventory Management (SIM) is imperative to set a balance between economic and environmental constraints. Sustainable-centric inventory model development under uncertainty fails to capture intricate demand patterns and fluctuating cost parameters. This has grounded the origin of Neutrosophic integrated inventory models based on three-valued logic of truth, indeterminacy, and falsity membership functions. This study reviews the state of the art of Neutrosophic integrated sustainable inventory models. The existing literature is explored in the three dimensions of inventory models handling defective and imperfect items, green models with carbon emissions and sustainable costs, and different modelling approaches. The advantages of blending Neutrosophic representations with sustainable inventory models are substantiated. The future directions of Neutrosophic-based models are also discussed in this study.

Keywords: Neutrosophy, Inventory models, Sustainability.

1 | Introduction

Inventory management is a significant component in supply chain operations, which serves as a tool in setting a balance between cost-effectiveness and resource consumption. In recent times, there has been a paradigm shift towards the development of sustainable-oriented inventory models. The industrial sectors of this age require inventory models to handle both the challenges of cost optimization and environmental conservation. The development of sustainable inventory models reflects the social responsibility of the industries, considering the economic and environmental parameters. The primary objective of these models is to address the challenges of global concerns such as carbon emissions, waste management, resource utilization, and regulatory compliance. Despite these advantages of Sustainable Inventory Management (SIM), the classical approach to model development falls short in handling the demand and cost uncertainties.

Corresponding Author: theebajoe@gmail.com
https://doi.org/10.22105/kmisj.v2i3.107

The sustainability component of the inventory management chiefly encompasses the aspects of handling defective or imperfect items, including the green costs parameters or any other measures of conserving the environment. The complexities in modelling these sustainable parameters shall be well tackled with soft computing techniques. Among them, Neutrosophic representations are characterized as a robust and versatile tool in inventory modelling. Smarandache introduced the theory of neutrosophy as a three-valued logic of the form (T, I, F) with truth, indeterminacy, and falsity functions. Neutrosophic Sets (NS) are defined as the generalized versions of fuzzy and intuitionistic fuzzy representations. NS is very competent in handling a high degree of ambiguity, vagueness, imprecision, and indeterminacy. This makes NSs more robust and suitable for modelling sustainable-centric inventory models. Researchers presented a survey on inventory models with Neutrosophic principles. The Neutrosophic inventory models exhibited more efficiency and competency in comparison with the fuzzy models. Researchers have also developed Neutrosophic integrated SIM models to accommodate the measures and methods adopted in promoting sustainable industrial sectors. The increasing development of SIM models under a Neutrosophic environment is the grounding factor for exploring the review of existing models. This review explores the state of the art of the existing Neutrosophic-based SIM models in the following three dimensions:

I. Neutrosophic SIM models focusing on defective and imperfect items.

II. Green Neutrosophic and eco-centered SIM models considering sustainable cost parameters.

III. Methodological approaches in building Neutrosophic SIM models.

SIM models with the integration of Neutrosophic logic demonstrate enhanced efficiency in capturing indeterminacies; improved optimization accuracy in designing solutions to sustainable challenges of industrial sectors. Furthermore, this study also presents the theoretical developments, contributions, and future direction of Neutrosophic SIM models.

2 | Theoretical Conceptualization of Neutrosophic Sets

This section presents the basic definitions and preliminaries associated with NSs.

2.1 | Neutrosophic Set

A Neutrosophic set is characterized independently by a truth-membership function $\alpha(x)$, an indeterminacy-membership function $\beta(x)$, and a falsity-membership function $\gamma(x)$, and each of the functions is defined from $X \rightarrow [0,1]$.

2.2 | Single-Valued Neutrosophic Set

I. Single valued NSs [1]

Let X be a universe of discourse. A Single Valued Neutrosophic Set (SVNS) Bover X is an object having the form:

$$B = \{\langle y, T_B(y), I_B(y)), F_B(y)\rangle: y\epsilon X\},$$

where $T_B : X \rightarrow [0,1], I_B : X \rightarrow [0,1]$, and $F_B : X \rightarrow [0,1]$ with the condition

$$0 \leq T_B(y) + I_B(y)) + F_B(y) \leq 3, \text{for all } y \in X.$$

The number $T_B(y), I_B(y)$ and $F_B(y)$ denote the degree of truth - membership, indeterminacy - membership, and falsity - membership of y to X, respectively.

II. Single valued NS [2]

An SVNS is denoted and defined as $\widetilde{A_N} = \{x, T_{\widetilde{A_N}}(x), I_{\widetilde{A_N}}(x), F_{\widetilde{A_N}}(x)/x\epsilon X\}$, where for each generic point x in X, $T_{\widetilde{A_N}}(x)$ called the truth membership function, $I_{\widetilde{A_N}}(x)$ called the indeterminacy membership function and

$F_{\widetilde{A_N}}(x)$ called the falsity membership function in [0,1] and $0 \leq T_{\widetilde{A_N}}(x) + I_{\widetilde{A_N}}(x) + F_{\widetilde{A_N}}(x) \leq 3$. For continuous SNVS $\widetilde{A_N} = \int_{A_N} \langle T_{\widetilde{A_N}}(x), I_{\widetilde{A_N}}(x), F_{\widetilde{A_N}}(x) \rangle / x_i, x_i \epsilon X$.

III. Single valued NS

An SVNS A on a universal set X is defined as $\tilde{a} = \langle T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} \rangle A = \{\langle T_A(x), I_A(x), F_A(x) \rangle : x \in X\}$, where $T_A, I_A, F_A : X \to [0,1]$, represents the degree of membership, degree of indeterminacy, and degree of non-membership, respectively, of the element $x \in X$, such that $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3$.

2.3 | Interval – Valued Neutrosophic Set

Let X be a nonempty set. Then an interval – valued Neutrosophic set [IVNS] $\widetilde{A_N}^{IV}$ of X is defined as [2]:

$$\widetilde{A_N}^{IV} = \left\{ \langle x; \left[T^L_{\widetilde{A_N}}, T^U_{\widetilde{A_N}}\right], \left[I^L_{\widetilde{A_N}}, I^U_{\widetilde{A_N}}\right], \left[F^L_{\widetilde{A_N}}, F^U_{\widetilde{A_N}}\right] \rangle : x\epsilon X \right\},$$

where

$$\left[T^L_{\widetilde{A_N}}, T^U_{\widetilde{A_N}}\right], \left[I^L_{\widetilde{A_N}}, I^U_{\widetilde{A_N}}\right] \text{and} \left[F^L_{\widetilde{A_N}}, F^U_{\widetilde{A_N}}\right] \subset [0,1] \text{ for all } x\epsilon X \; T^L_{\widetilde{A_N}} = \inf\left(T_{\widetilde{A_N}}\right), T^U_{\widetilde{A_N}} = \sup\left(T_{\widetilde{A_N}}\right); I^L_{\widetilde{A_N}} = \inf\left(I_{\widetilde{A_N}}\right),$$

$$I^U_{\widetilde{A_N}} = \sup\left(I_{\widetilde{A_N}}\right); \text{and } F^L_{\widetilde{A_N}} = \inf\left(F_{\widetilde{A_N}}\right), F^U_{\widetilde{A_N}} = \sup\left(F_{\widetilde{A_N}}\right).$$

2.4 | Operations of Neutrosophic Sets

Let A and B be two simplified NSs, then the operations of the NSs are [1]:

$$A + B = \{\langle X, T_A(x) + T_B(y) - T_A(x).T_B(y), I_A(x) + I_B(y) - I_A(x).I_B(y), F_A(x) + F_B(y) - F_A(x).F_B(y) | \; x, y \in X \rangle\},$$

$$A \times B = \{\langle X, T_A(x).T_B(y), I_A(x).I_B(y) \; F_A(x).F_B(y) | x, y \in X \rangle\},$$

$$\lambda.A = \{\langle X, 1 - \left(1 - T_A{}^{(x)}\right)^\lambda, \left(1 - T_A{}^{(x)}\right)^\lambda, \left(1 - T_A{}^{(x)}\right)^\lambda | x \in X \rangle\}, \lambda > 0,$$

$$A^\lambda = \{\langle x, T_A{}^\lambda(x), I_A{}^\lambda(x), F_A{}^\lambda(x) | x \in X \rangle\}, \lambda > 0.$$

2.5 | Neutrosophic Number

Let x be a generic element of a non-empty set. A Neutrosophic number $\widetilde{A_N}$ in X is defined as $\widetilde{A_N} = \{x, \langle T_{\widetilde{A_N}}(x), I_{\widetilde{A_N}}(x), F_{\widetilde{A_N}}(x) \rangle / x\epsilon X\}$, for all $T_{\widetilde{A_N}}(x), I_{\widetilde{A_N}}(x)$ and $F_{\widetilde{A_N}}(x)$ belongs to $]0^-, 1^+[$ where $T_{\widetilde{A_N}}: X \to]0^-, 1^+[I,_{\widetilde{A_N}} : X \to]0^-, 1^+[$ and $F_{\widetilde{A_N}}: X \to]0^-, 1^+[$ are functions of truth – membership, indeterminacy membership, and falsity, membership in $\widetilde{A_N}$ respectively with [2]:

$$0^- \leq T_{\widetilde{A_N}}(x) + I_{\widetilde{A_N}}(x) + F_{\widetilde{A_N}}(x) \leq 3^+.$$

2.6 | Single-Valued Neutrosophic Number

A Single Valued Neutrosophic Number (SVNN), in the set of real numbers R with truth – membership function $T_{\tilde{a}}$, indeterminacy-membership function $I_{\tilde{a}}$ and falsity-membership function $F_{\tilde{a}}$, is defined as

$$T_{\tilde{a}}(x) = \begin{cases} f_{\tilde{a}}(x), \text{if } a_1 \leq x < b_{1,} \\ 1, \quad \text{if } b_1 \leq x < c_1, \\ g_{\tilde{a}}(x), \text{if } c_1 \leq x < d_1, \\ 0, \quad \text{otherwise,} \end{cases}$$

$$I_{\tilde{a}}(x) = \begin{cases} l_{\tilde{a}}(x), \text{if } a_2 \le x < b_2, \\ 1, \quad \text{if } b_2 \le x < c_2, \\ m_{\tilde{a}}(x), \text{if } c_2 \le x < d_2, \\ 0, \quad \text{otherwise}, \end{cases}$$

and

$$T_{\tilde{a}}(x) = \begin{cases} h_{\tilde{a}}(x), \text{if } a_3 \le x < b_3, \\ 1, \quad \text{if } b_3 \le x < c_3, \\ k_{\tilde{a}}(x), \text{if } c_3 \le x < d_3 \\ 0, \quad \text{otherwise}. \end{cases}$$

Respectively, where $0 \le T_{\tilde{a}} + I_{\tilde{a}} + F_{\tilde{a}} \le 3$ and $a_i, b_i, c_i, d_i \mathbb{R}$.

$a_i \le b_i \le c_i \le d_i$ where $i = 1,2,3$ and the function $f_{\tilde{a}},, g_{\tilde{a}}, l_{\tilde{a}}, m_{\tilde{a}}, h_{\tilde{a}}, k_{\tilde{a}}: \mathbb{R} \to [0,1]$. The function $f_{\tilde{a}}, m_{\tilde{a}}, k_{\tilde{a}}$ are non-decreasing continuous functions and $g_{\tilde{a}}, l_{\tilde{a}}, h_{\tilde{a}}$ are non –increasing continuous functions. SVNN is also denoted by $\tilde{a} = \langle (a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2), (a_3, b_3, c_3, d_3) \rangle$.

2.7 | Triangular Neutrosophic Number

A Triangular Neutrosophic Number (TNN) is denoted by $\overline{V} = \langle (p^l, p^m, p^n), (\alpha^p, \alpha^q, \alpha^r) \rangle$ whose three membership functions for the truth, indeterminacy, and falsity of x can be defined as follows [3]:

$$\alpha^p(x) = \begin{cases} \frac{(x - p^l)}{(p^m - p^n)} \alpha^p, & p^l \le x < p^m, \\ \alpha^p, & x = p^l, \\ \frac{(p^n - x)}{(p^n - p^m)} \alpha^p, & p^m \le x < p^n, \\ 0, & \text{otherwise}, \end{cases}$$

$$\alpha^q(x) = \begin{cases} \frac{(p^m - x)}{(p^m - p^n)} \alpha^q, & p^l \le x < p^m, \\ \alpha^q, & x = p^m, \\ \frac{(x - p^n)}{(p^n - p^m)} \alpha^q, & p^m \le x < p^n \\ 1, & \text{otherwise}, \end{cases}$$

$$\alpha^r(x) = \begin{cases} \frac{(p^l - x)}{(p^m - p^n)} \alpha^r, & p^l \le x < p^m, \\ \alpha^r, & x = p^m, \\ \frac{(x - p^n)}{(p^n - p^m)} \alpha^r, & p^m \le x < p^n, \\ 1, & \text{otherwise}, \end{cases}$$

where $0 \le \alpha^p(x) + \alpha^q(x) + \alpha^r(x) \le 3, x \in R$. Accordingly, when $p^l \ge 0$, R is called a nonnegative TNN. Similarly, when $p^l < 0$, R becomes a negative TNN.

2.8 | Triangular Single -Valued Neutrosophic Number

A triangular single- valued Neutrosophic number $(\tilde{S})$ is defined as $(\tilde{S}) = \langle (m_1, m_2, m_3; n_1, n_2, n_3; p_1, p_2, p_3) \rangle$. Here, $\pi_{\tilde{S}}: R \to [0,1]$ is the truth membership function, $\theta_{\tilde{S}}: R \to [0,1]$ is the hesitation membership function, and the falsity membership function is mathematically and graphically (see *Fig. 1*) defined as follows [4]:

$$\pi_{\tilde{S}}(x)=\begin{cases}\dfrac{x-m_1}{m_2-m_1}, & \text{for } m_1 \le x < m_1,\\ 1, & \text{for } x=m_2,\\ \dfrac{m_3-x}{m_3-m_2}, & \text{for } m_2 < x \le m_3,\\ 0, & \text{otherwise,}\end{cases}$$

$$\pi_{\tilde{S}}(x)=\begin{cases}\dfrac{x-m_1}{m_2-m_1}, & \text{for } m_1 \le x < m_1,\\ 1, & \text{for } x=m_2,\\ \dfrac{m_3-x}{m_3-m_2}, & \text{for } m_2 < x \le m_3,\\ 0, & \text{otherwise,}\end{cases}$$

$$\pi_{\tilde{S}}(x)=\begin{cases}\dfrac{x-m_1}{m_2-m_1}, & \text{for } m_1 \le x < m_1,\\ 1, & \text{for } x=m_2,\\ \dfrac{m_3-x}{m_3-m_2}, & \text{for } m_2 < x \le m_3,\\ 0, & \text{otherwise.}\end{cases}$$

2.9 | Neutrosophic Trapezoidal Interval Valued Number

A single-valued trapezoidal Neutrosophic number $\tilde{a}=\langle(a_1,b_1,c_1,d_1);w_{\tilde{a}},u_{\tilde{a}},y_{\tilde{a}}\rangle$ is a special Neutrosophic set on the real number set R, whose truth – membership, indeterminacy–membership, and falsity–membership are given as follows [5]:

$$\mu_{\tilde{a}}(x)=\begin{cases}\dfrac{(x-a_1)w_{\tilde{a}}}{b_1-a_1}, & (a_1 \le x < b_1),\\ w_{\tilde{a}}, & (b_1 \le x \le c_1),\\ \dfrac{(d_1-x)w_{\tilde{a}}}{d_1-c_1}, & (c_1 < x \le d_1),\\ 0, & \text{otherwise,}\end{cases}$$

and

$$\lambda_{\tilde{a}}(x)=\begin{cases}(b_1-x+y_{\tilde{a}}(x-a_1))/(b_1-a_1), & (a_1 \le x < b_1),\\ y_{\tilde{a}}, & (b_1 \le x \le c_1),\\ \dfrac{x-c_1+y_{\tilde{a}}(d_1-x)}{(d_1-c_1)}, & (c_1 < x \le d_1),\\ 1, & \text{otherwise.}\end{cases}$$

respectively.

2.10 | Single - Valued Pentagonal Neutrosophic Number

A Single-Valued Pentagonal Neutrosophic Number $(\widetilde{M})$ is defined as, $(\widetilde{S}s)=$ $\langle[(s^1,t^1,u^1,v^1,w^1);\mu],[(s^2,t^2,u^2,v^2,w^2);\theta],[(s^3,t^3,u^3,v^3,w^3);\eta]\rangle$, where $\mu,\theta,\eta\epsilon[0,1]$.

The truthness function $(\mu_{\widetilde{S}s})\colon \mathbb{R}\to[0,\mu]$, the indeterminacy function $(\theta_{\widetilde{S}s})\colon \mathbb{R}\to[\theta,1]$, and the falsity function $(\eta_{\widetilde{S}s})\colon \mathbb{R}\to[\eta,1]$ are given as [6]:

$$\mu_{\widetilde{ss}} = \begin{cases} \mu_{\widetilde{ssl_1}}(x), & s^1 \le x < t^1, \\ \mu_{\widetilde{ssl_2}}(x), & t^1 \le x < u^1, \\ \mu, & x = u^1, \\ \mu_{\widetilde{sr2}}(x), & u^1 \le x < v^1, \\ \mu_{\widetilde{ssr_1}}(x), & v^1 \le x < w^1, \\ 0, & \text{otherwise}, \end{cases}$$

$$\theta_{\widetilde{ss}} = \begin{cases} \theta_{\widetilde{ssl_1}}(x), & s^2 \le x < t^2, \\ \theta_{\widetilde{ssl_2}}(x), & t^2 \le x < u^2, \\ \theta, & x = u^2, \\ \theta_{\widetilde{sr2}}(x), & u^2 \le x < v^2, \\ \theta_{\widetilde{ssr_1}}(x), & v^2 \le x < w^2, \\ 1, & \text{otherwise}, \end{cases}$$

$$\eta_{\widetilde{ss}} = \begin{cases} \eta_{\widetilde{ssl_1}}(x), & s^3 \le x < t^3, \\ \eta_{\widetilde{ssl_2}}(x), & t^3 \le x < u^3, \\ \eta, & x = u^3, \\ \eta_{\widetilde{sr2}}(x), & u^3 \le x < v^3, \\ \eta_{\widetilde{ssr_1}}(x), & v^3 \le x < w^{3,} \\ 1, & \text{otherwise}. \end{cases}$$

2.11 | Generalized Single-Valued Neutrosophic Number

A single-valued Neutrosophic Number defined on the set of real numbers ℝ is said to be a Generalized Single Valued Neutrosophic Number (GSVNN) $G_{\tilde{a}} = \langle T_{G_{\tilde{a}}}, I_{G_{\tilde{a}}}, F_{G_{\tilde{a}}}; \omega, \rho, \delta \rangle$, with truth-membership function $T_{G_{\tilde{a}}}(x)$, indeterminacy-membership function $I_{G_{\tilde{a}}}(x)$ and falsity-membership function $F_{G_{\tilde{a}}}(x)$ has the following Characteristics:

I. $T_{G_{\tilde{a}}}, I_{G_{\tilde{a}}}, F_{G_{\tilde{a}}}\ \mathbb{R} \to [0,1]$.

II. $T_{G_{\tilde{a}}} = 0, I_{G_{\tilde{a}}} = 1, F_{G_{\tilde{a}}} = 1$ for all $x \in (-\infty, a_i] \cup [d_i, \infty)$.

III. $T_{G_{\tilde{a}}}(x)$ is strictly increasing on $[a_1, b_1]$ and $T_{G_{\tilde{a}}}(x)$ is strictly decreasing on $[c_1, d_1]$.

$I_{G_{\tilde{a}}}(x)$ is strictly decreasing on $[a_2, b_2]$ and $T_{G_{\tilde{a}}}(x)$ is strictly decreasing on $[c_2, d_2]$.

$F_{G_{\tilde{a}}}(x)$ is strictly decreasing on $[a_3, b_3]$ and $T_{G_{\tilde{a}}}(x)$ is strictly increasing on $[c_3, d_3]$.

IV. $T_{G_{\tilde{a}}}(x) = \omega$ for all $x \epsilon [b_1, c_1]$ where $0 < \omega \le 1$. $I_{G_{\tilde{a}}}(x) = \rho$ for all $x \epsilon [b_2, c_2]$ where $0 \le \rho < 1$. $F_{G_{\tilde{a}}}(x) = \delta$ for all $x \epsilon [b_2, c_2]$ where $0 \le \delta < 1$.

2.12 | A Generalized Parabolic Single-Valued Neutrosophic Number

A Generalized Parabolic Single-Valued Neutrosophic Number (GPSVNN), $\widetilde{A} = \langle (T_{\widetilde{A}}; \omega), (I_{\widetilde{A}}; \rho), (F_{\widetilde{A}}; \delta) \rangle$, is a Neutrosophic set on the real number ℝ with truth membership function $T_{\widetilde{A}}$, indeterminacy-membership function $I_{\widetilde{A}}$, and falsity-membership function $F_{\widetilde{A}}$, is defined as

$$T_{\widetilde{A}}(x) = \begin{cases} \omega\left(\dfrac{x - a_1}{b_1 - a_1}\right)^2, & x \epsilon [a_1, b_1), \\ \omega, & x \epsilon [b_1, c_1), \\ \omega\left(\dfrac{d_1 - x}{d_1 - c_1}\right)^2, & x \epsilon [c_1, d_1), \\ 0, & \text{otherwise}, \end{cases}$$

$$I_{\tilde{A}}(x) = \begin{cases} 1 - \left(\frac{x - a_2}{b_2 - a_2}\right)^2 (1 - \rho), & x\epsilon[a_2, b_2), \\ \rho, & x\epsilon[b_2, c_2), \\ 1 - \left(\frac{d_2 - x}{d_2 - c_2}\right)^2 (1 - \rho), & x\epsilon[c_2, d_2), \\ 1, & \text{otherwise}, \end{cases}$$

$$F_{\tilde{A}}(x) = \begin{cases} 1 - \left(\frac{x - a_3}{b_3 - a_3}\right)^2 (1 - \delta), & x\epsilon[a_3, b_3), \\ \delta, & x\epsilon[b_3, c_3), \\ 1 - \left(\frac{d_1 - x}{d_1 - c_1}\right)^2, & x\epsilon[c_3, d_3), \\ 1, & \text{otherwise}, \end{cases}$$

where, $0 \leq T_{\tilde{A}} + I_{\tilde{A}} + F_{\tilde{A}} \leq 3, 0 < \omega \leq 1, 0 \leq \rho < 1, 0 \leq \delta < 1$ and $a_i, b_i, c_i, d_i \in \mathbb{R}$,

$a_i \leq b_i \leq c_i \leq d_i$ where $i = 1,2,3$.

2.13 | Generalized Triangular Neutrosophic Number

A generalized TNN $(\tilde{N}) = \langle (m_1, m_2, m_3; \mu), (n_1, n_2, n_3, n_4; \nu), (p_1, p_2, p_3, p_4; \zeta) \rangle$ where $\mu, \nu, \zeta \in [0,1]$. Here, $\pi_{\tilde{N}}: R \rightarrow [0, \mu]$ is the truth membership function, $\theta_{\tilde{N}}: R \rightarrow [\nu, 1]$ is the hesitation membership function, and the falsity membership function is $\eta_{\tilde{N}}: R \rightarrow [\zeta, 1]$, where the membership functions are mathematically defined as follows [7]:

$$\pi_{\tilde{N}}(x) \begin{cases} \frac{(x - m_1)}{(m_2 - m_1)} \mu & \text{if } m_1 \leq x < m_2, \\ \mu & \text{if } x = m_2, \\ \frac{(m_3 - x)}{(m_3 - m_2)} \mu & \text{if } m_2 < x \leq m_3, \\ 0 & \text{otherwise}, \end{cases}$$

$$\theta_{\tilde{N}}(x) \begin{cases} \frac{(n_2 - x) + \nu(x - n_1)}{(n_2 - n_1)} & \text{if } n_1 \leq x < n_2, \\ \nu & \text{if } x = n_2, \\ \frac{(x - n_2) + V(n_3 - x)}{(n_3 - n_2)} \mu & \text{if } n_2 < x \leq n_3, \\ 1 & \text{otherwise}, \end{cases}$$

$$\eta_{\tilde{N}}(x) = \begin{cases} \frac{(p_2 - x) + \zeta(x - p_1)}{(p_2 - p_1)} \mu & \text{if } p_1 \leq x < p_2, \\ \zeta & \text{if } x = p_2, \\ \frac{(x - p_2) + (x - p_1)}{(p_3 - p_2)} \mu & \text{if } p_2 < x \leq p_3, \\ 1 & \text{otherwise}. \end{cases}$$

2.14 | Single – Valued Trapezoidal Neutrosophic Number

A Single-Valued Trapezoidal Neutrosphic Number (SVTpN) of type I, denoted as $\tilde{G}_{TpN} = (i_1, i_2, i_3, i_4; k_1, k_2, k_3, k_4; l_1, l_2, l_3, l_4)$ is a specific type of Neutrosophic set on the real number $\mathbb{R}$. It is characterized by its truth, indeterminacy, and falsity membership functions, which are defined as follows [8]:

$$T_{\widetilde{G}_{TpN}}(\zeta)=\begin{cases}\dfrac{\zeta-i_1}{i_2-i_1}, & \text{when } i_1\le\zeta<i_2,\\ 1, & \text{when } i_2\le\zeta\le i_3,\\ \dfrac{i_4-\zeta}{i_4-i_3}, & \text{when } i_3<\zeta\le i_4,\\ 0, & \text{otherwise,}\end{cases}$$

$$I_{\widetilde{G}_{TpN}}(\zeta)=\begin{cases}\dfrac{k_2-\zeta}{k_2-k_1}, & \text{when } k_1\le\zeta<k_2,\\ 0, & \text{when } k_2\le\zeta\le k_3,\\ \dfrac{\zeta-k_2}{k_4-k_3}, & \text{when } k_3<\zeta\le k_4,\\ 1, & \text{otherwise,}\end{cases}$$

$$F_{\widetilde{G}_{TpN}}(\zeta)=\begin{cases}\dfrac{l_2-\zeta}{l_2-l_1}, & \text{when } l_1\le\zeta<l_2,\\ 0, & \text{when } l_2\le\zeta\le l_3,\\ \dfrac{\zeta-l_3}{l_4-l_3}, & \text{when } i_3<\zeta\le i_4,\\ 1, & \text{otherwise.}\end{cases}$$

2.15 | Operations of Single-Valued Neutrosophic Number

2.15.1 | Addition of single-valued neutrosophic number

Let $A_1=(a_1,b_1,c_1)$ and $A_2=(a_2,b_2,c_2)$ be two SVN numbers, then the summation between A_1 and A_2 is defined as follows:

$$A_1\oplus A_2=(a_1+a_2-a_1a_2,b_1b_2,c_1c_2).$$

2.15.2 | Multiplication of single-valued neutrosophic numbers

Let $A_1=(a_1,b_1,c_1)$ and $A_2=(a_2,b_2,c_2)$ be two SVN numbers, then multiplication between A_1 and A_2 is defined as follows:

$$A_1\otimes A_2=(a_1a_2,b_1+b_2-b_1b_2,c_1+c_2-c_1c_2).$$

2.15.3 | Scalar multiplication of single-valued neutrosophic numbers

Let $A=(a,b,c)$ be an SVN number and $\lambda\in\mathbb{R}$ an arbitrary positive real number, then

$$\lambda A=\left(1-(1-a)^{\lambda},b^{\lambda},c^{\lambda}\right),\lambda>0.$$

2.15.4 | Operations of triangular neutrosophic number

Let $A_1=\langle(p^l,p^m,p^n),(\alpha^p,\alpha^q,\alpha^r)\rangle$ and $\langle(q^l,q^m,q^n),(\beta^p,\beta^q,\beta^r)\rangle$ be two TNNs. Then the arithmetic relations are defined as follows [3]:

I. $A_1\oplus A_2=\langle(p^l+q^l,p^m+q^m,p^n+q^n),(\alpha^p\wedge\beta^p,\alpha^q\wedge\beta^q,\alpha^r\wedge\beta^r)\rangle$.

II. $A_1-A_2=\langle(p^l-q^n,p^m-q^m,p^n-q^l),(\alpha^p\wedge\beta^p,\alpha^q\wedge\beta^q,\alpha^r\wedge\beta^r)\rangle$.

III. $A_1\otimes A_2=\langle(p^lq^l,p^mq^m,p^nq^n),(\alpha^p\wedge\beta^p,\alpha^q\wedge\beta^q,\alpha^r\wedge\beta^r)\rangle$, if $p^l>0,q^l>0$.

$\langle(p^lq^n,p^mq^m,p^nq^l),(\alpha^p\wedge\beta^p,\alpha^q\wedge\beta^q,\alpha^r\wedge\beta^r)\rangle$, if $p^l<0,q^l>0$.

$\langle(p^n q^n, p^m q^m, p^l q^l), (\alpha^p \wedge \beta^p, \alpha^q \wedge \beta^q, \alpha^r \wedge \beta^r)\rangle$, if $p^l < 0, q^l < 0$.

IV. $\lambda A_1 = \langle(\lambda p^l, \lambda p^m, \lambda p^n), (\alpha^p, \alpha^q, \alpha^r)\rangle$, if $\lambda > 0$.

$\lambda A_1 = \langle(\lambda p^n, \lambda p^m, \lambda p^l), (\alpha^p, \alpha^q, \alpha^r)\rangle$, if $\lambda < 0$.

V. $A_1/A_2 = \langle(p^l/q^n, p^m/q^m, p^n/q^l), (\alpha^p \wedge \beta^p, \alpha^q \wedge \beta^q, \alpha^r \wedge \beta^r)\rangle$, if $p^n > 0, q^n > 0$.

$\langle(p^n/q^n, p^m/q^m, p^l/q^l), (\alpha^p \wedge \beta^p, \alpha^q \wedge \beta^q, \alpha^r \wedge \beta^r)\rangle$, if $p^n < 0, q^n > 0$,

$\langle(p^n/q^l, p^m/q^m, p^l/q^n), (\alpha^p \wedge \beta^p, \alpha^q \wedge \beta^q, \alpha^r \wedge \beta^r)\rangle$, if $p^n < 0, q^n < 0$.

2.16 | Operational Laws on IVTrNeNs

Let $\widetilde{a_{N_1}}^{IV} = \langle[\{(a_1, b_1, c_1, d_1), u_{\widetilde{a_{N_1}}^{IV}}\}, \{(e_1, f_1, g_1, h_1), v_{\widetilde{a_{N_1}}^{IV}}\}, \{(l_1, m_1, n_1, p_1), w_{\widetilde{a_{N_1}}^{IV}}\}]\rangle$ and $\widetilde{a_{N_2}}^{IV} = \langle[\{(a_2, b_2, c_2, d_2), u_{\widetilde{a_{N_2}}^{IV}}\}, \{(e_2, f_2, g_2, h_2), v_{\widetilde{a_{N_2}}^{IV}}\}, \{(l_2, m_2, n_2, p_2), w_{\widetilde{a_{N_2}}^{IV}}\}]\rangle$ be two IVTrNeNs with twelve components, where $u_{\widetilde{a_{N_1}}^{IV}} = [u_{\widetilde{a_{N_1}}^{IV}}{}^L, u_{\widetilde{a_{N_1}}^{IV}}{}^U]$, $u_{\widetilde{a_{N_2}}^{IV}} = [u_{\widetilde{a_{N_2}}^{IV}}{}^L, u_{\widetilde{a_{N_2}}^{IV}}{}^U]$, $v_{\widetilde{a_{N_1}}^{IV}} = [v_{\widetilde{a_{N_1}}^{IV}}{}^L, v_{\widetilde{a_{N_1}}^{IV}}{}^U]$, $v_{\widetilde{a_{N_2}}^{IV}} = [v_{\widetilde{a_{N_2}}^{IV}}{}^L, v_{\widetilde{a_{N_2}}^{IV}}{}^U]$, $w_{\widetilde{a_{N_1}}^{IV}} = [w_{\widetilde{a_{N_1}}^{IV}}{}^L, w_{\widetilde{a_{N_1}}^{IV}}{}^U]$, $w_{\widetilde{a_{N_2}}^{IV}} = [w_{\widetilde{a_{N_2}}^{IV}}{}^L, w_{\widetilde{a_{N_2}}^{IV}}{}^U]$ then the following operations hold [2].

2.16.1 | Addition of IVTrNeNs

$$\widetilde{a_{N_1}}^{IV} + \widetilde{a_{N_2}}^{IV} = \left\langle \begin{bmatrix} \{(a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2); u_{\widetilde{a_{N_1}}^{IV}} \wedge u_{\widetilde{a_2}^{IV}}\} \\ \{(e_1 + e_2, f_1 + f_2, g_1 + g_2, h_1 + h_2);\ v_{\widetilde{a_{N_1}}^{IV}} \vee v_{\widetilde{a_{N_2}}^{IV}} \\ \{(l_1 + l_2, m_1 + m_2, n_1 + n_2, p_1 + p_2);\ w_{\widetilde{a_{N_1}}^{IV}} \vee w_{\widetilde{a_{N_2}}^{IV}} \end{bmatrix} \right\rangle.$$

2.16.2 | Negative of IVTrNeNs

$$-\widetilde{a_{N_2}}^{IV} = \left\langle \begin{matrix} [\{-d_2, -c_2, -b_2, -a_2), u_{\widetilde{a_{N_2}}^{IV}}\}, \{-h_2, -g_2, -f_2, -e_2), v_{\widetilde{a_{N_2}}^{IV}}\}, \\ \{-p_2, -n_2, -m_2, -l_2), w_{\widetilde{a_{N_2}}^{IV}}\}] \end{matrix} \right\rangle.$$

2.16.3 | Subtraction of IVTrNeNs

$$\widetilde{a_{N_1}}^{IV} - \widetilde{a_{N_2}}^{IV} = \left\langle \begin{bmatrix} \{(a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2); u_{\widetilde{a_{N_1}}^{IV}} \wedge u_{\widetilde{a_2}^{IV}}\} \\ \{(e_1 - h_2, f_1 - g_2, g_1 - f_2, h_1 - e_2);\ v_{\widetilde{a_{N_1}}^{IV}} \vee v_{\widetilde{a_{N_2}}^{IV}} \\ \{(l_1 - p_2, m_1 - n_2, n_1 - m_2, p_1 - l_2);\ w_{\widetilde{a_{N_1}}^{IV}} \vee w_{\widetilde{a_{N_2}}^{IV}} \end{bmatrix} \right\rangle.$$

2.16.4 | Scalar multiplication of SVTrNeN

$$\lambda \widetilde{a_{N_1}}^{IV} = \begin{cases} \langle\{(\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1); u_{\widetilde{a_{N_1}}^{IV}}\}, \{(\lambda e_1, \lambda f_1, \lambda g_1, \lambda h_1); v_{\widetilde{a_{N_1}}^{IV}}\}, \{(\lambda l_1, \lambda m_1, \lambda n_1, \lambda p_1); w_{\widetilde{a_{N_1}}^{IV}}\}\rangle \text{ if } \lambda > 0. \\ \langle\{(\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1); u_{\widetilde{a_{N_1}}^{IV}}\}, \{(\lambda e_1, \lambda f_1, \lambda g_1, \lambda h_1); v_{\widetilde{a_{N_1}}^{IV}}\}, \{(\lambda e_1, \lambda f_1, \lambda g_1, \lambda h_1); w_{\widetilde{a_{N_1}}^{IV}}\}\rangle \text{ if } \lambda < 0. \end{cases}$$

2.16.5 | Multiplication of SVTrNeN

$$\widetilde{a_{N_1}}^{IV}.\widetilde{a_{N_2}}^{IV} = \begin{cases} \langle(a_1.a_2, b_1.b_2, c_1.c_2, d_1.d_2); (e_1.e_2, f_1.f_2, g_1.g_2, h_1.h_2); (l_1.l_2, m_1.m_2, n_1.n_2, p_1.p_2) \\ \left[u_{\widetilde{a_{N_1}}^{IV}} \wedge u_{\widetilde{a_2}^{IV}}\right], \left[v_{\widetilde{a_{N_1}}^{IV}} \vee v_{\widetilde{a_2}^{IV}}\right], \left[u_{\widetilde{a_{N_1}}^{IV}} \vee u_{\widetilde{a_2}^{IV}}\right] \text{if} d_1 > 0, d_2 > 0, h_1 > 0, h_2 > 0, p_1 > 0, p_2 > 0\rangle \\ \langle(a_1.d_2, b_1.c_2, c_1.b_2, d_1.a_2); (e_1.h_2, f_1.g_2, g_1.f_2, h_1.e_2); (l_1.p_2, m_1.n_2, n_1.m_2, p_1.l_2) \\ \left[u_{\widetilde{a_{N_1}}^{IV}} \wedge u_{\widetilde{a_2}^{IV}}\right], \left[v_{\widetilde{a_{N_1}}^{IV}} \vee v_{\widetilde{a_2}^{IV}}\right], \left[u_{\widetilde{a_{N_1}}^{IV}} \vee u_{\widetilde{a_2}^{IV}}\right] \text{if} d_1 < 0, d_2 > 0, h_1 < 0, h_2 > 0, p_1 < 0, p_2 > 0\rangle \\ (d_1.d_2, c_1.c_2, b_1.b_2, a_1.a_2); (h_1.h, g_1.g_2, f_1.f_2, e_1.e_2); (p_1.p_2, n_1.n_2, m_1.m_2, l_1.l_2) \\ \left\langle\left[u_{\widetilde{a_{N_1}}^{IV}} \wedge u_{\widetilde{a_2}^{IV}}\right], \left[v_{\widetilde{a_{N_1}}^{IV}} \vee v_{\widetilde{a_2}^{IV}}\right], \left[u_{\widetilde{a_{N_1}}^{IV}} \vee u_{\widetilde{a_2}^{IV}}\right] \text{if} d_1 < 0, d_2 < 0, h_1 < 0, h_2 < 0, p_1 < 0, p_2 < 0\right\rangle \end{cases}.$$

2.16.6 | Inverse of SVTrNeN

$$s(\widetilde{a_{N_1}}^{IV})^{-1} = \frac{1}{\widetilde{a_{N_1}}^{IV}} \begin{cases} \langle\left(\frac{1}{d_1}, \frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1}\right); \left(\frac{1}{h_1}, \frac{1}{g_1}, \frac{1}{f_1}, \frac{1}{e_1}\right); \left(\frac{1}{p_1}, \frac{1}{n}, \frac{1}{m_1}, \frac{1}{l_1}\right); u_{\widetilde{a_{N_1}}^{IV}}, v_{\widetilde{a_{N_1}}^{IV}}, w_{\widetilde{a_{N_1}}^{IV}}\rangle, \\ \text{if} a_1 > 0, b_1 > 0, c_1 > 0, d_1 > 0, e_1 > 0, f_1 > 0, g_1 > 0, h_1 > 0, l_1 > 0, m_1 > 0, n_1 > 0, p_1 > 0 \\ \langle\left(\frac{1}{a_1}, \frac{1}{b_1}, \frac{1}{c_1}, \frac{1}{d_1}\right); \left(\frac{1}{e_1}, \frac{1}{f_1}, \frac{1}{g_1}, \frac{1}{h_1}\right); \left(\frac{1}{l_1}, \frac{1}{m_1}, \frac{1}{n_1}, \frac{1}{p_1}\right); u_{\widetilde{a_{N_1}}^{IV}}, v_{\widetilde{a_{N_1}}^{IV}}, w_{\widetilde{a_{N_1}}^{IV}}\rangle \\ \text{if} a_1 < 0, b_1 < 0, c_1 < 0, d_1 < 0, e_1 < 0, f_1 < 0, g_1 < 0, h_1 < 0, l_1 < 0, m_1 < 0, n_1 < 0, p_1 < 0 \end{cases}.$$

2.16.7 | Division of SVTrNeN

$$\frac{\widetilde{a_{N_1}}^{IV}}{\widetilde{a_{N_2}}^{IV}} = \begin{cases} \langle\left(\frac{a_1}{d_1}, \frac{b_1}{c_1}, \frac{c_1}{b_1}, \frac{d_1}{a_1}\right), \left(\frac{e_1}{h_1}, \frac{f_1}{g_1}, \frac{g_1}{f_1}, \frac{h_1}{e_1}\right); \left(\frac{l_1}{p_1}, \frac{m}{n_1}, \frac{n_1}{m_1}, \frac{p_1}{l_1}\right); \left[u_{\widetilde{a_{N_1}}^{IV}} \wedge u_{\widetilde{a_2}^{IV}}\right], \left[v_{\widetilde{a_{N_1}}^{IV}} \vee v_{\widetilde{a_{N_2}}^{IV}}\right], \left[w_{\widetilde{a_{N_1}}^{IV}} \vee w_{\widetilde{a_{N_2}}^{IV}}\right]\rangle, \\ \text{if} d_1 > 0, d_2 > 0, h_1 > 0, h_2 > 0, p_1 > 0, p_2 > 0, \\ \langle\left(\frac{d_2}{d_1}, \frac{c_2}{c_1}, \frac{b_2}{b_1}, \frac{a_2}{a_1}\right), \left(\frac{h_2}{h_1}, \frac{g_2}{g_1}, \frac{f_2}{f_1}, \frac{e_2}{e_1}\right); \left(\frac{p_2}{p_1}, \frac{n_2}{n_1}, \frac{m_2}{m_1}, \frac{l_2}{l_1}\right); \left[u_{\widetilde{a_{N_1}}^{IV}} \wedge u_{\widetilde{a_2}^{IV}}\right], \left[v_{\widetilde{a_{N_1}}^{IV}} \vee v_{\widetilde{a_{N_2}}^{IV}}\right], \left[w_{\widetilde{a_{N_1}}^{IV}} \vee w_{\widetilde{a_{N_2}}^{IV}}\right]\rangle, \text{if} \\ d_1 < 0, d_2 > 0, h_1 < 0, h_2 > 0, p_1 < 0, p_2 > 0, \\ \langle\left(\frac{d_2}{a_1}, \frac{c_2}{b_1}, \frac{b_2}{c_1}, \frac{a_2}{d_1}\right), \left(\frac{h_2}{e_1}, \frac{g_2}{f_1}, \frac{f_2}{g_1}, \frac{e_2}{h_1}\right); \left(\frac{p_2}{l_1}, \frac{n_2}{m_1}, \frac{m_2}{n_1}, \frac{l_2}{p_1}\right); \left[u_{\widetilde{a_{N_1}}^{IV}} \wedge u_{\widetilde{a_2}^{IV}}\right], \left[v_{\widetilde{a_{N_1}}^{IV}} \vee v_{\widetilde{a_{N_2}}^{IV}}\right], \left[w_{\widetilde{a_{N_1}}^{IV}} \vee w_{\widetilde{a_{N_2}}^{IV}}\right]\rangle, \text{if} \\ d_1 < 0, d_2 < 0, h_1 < 0, h_2 < 0, p_1 < 0, p_2 < 0. \end{cases},$$

where $u_{\widetilde{a_{N_1}}^{IV}} \wedge u_{\widetilde{a_2}^{IV}} = \left[\min\left(u_{\widetilde{a_{N_1}}^{IV}}{}^L, u_{\widetilde{a_{N_2}}^{IV}}{}^L\right), \min\left(u_{\widetilde{a_{N_1}}^{IV}}{}^U, u_{\widetilde{a_{N_2}}^{IV}}{}^U\right)\right]$, $v_{\widetilde{a_{N_1}}^{IV}} \vee v_{\widetilde{a_{N_2}}^{IV}} = [\max\left(v_{\widetilde{a_{N_1}}^{IV}}{}^L, v_{\widetilde{a_{N_2}}^{IV}}{}^L\right), \max(v_{\widetilde{a_{N_1}}^{IV}}{}^U, v_{\widetilde{a_{N_2}}^{IV}}{}^U)$ and $w_{\widetilde{a_{N_1}}^{IV}} \vee w_{\widetilde{a_{N_2}}^{IV}} = [\max\left(w_{\widetilde{a_{N_1}}^{IV}}{}^L, w_{\widetilde{a_{N_1}}^{IV}}{}^L\right), \max(w_{\widetilde{a_{N_1}}^{IV}}{}^U, w_{\widetilde{a_{N_1}}^{IV}}{}^U)]$.

2.17 | Operations on Pentagonal Neutrosophic Number

Let $A = \langle(s^1, t^1, u^1 v^1, w^1); \mu_A, \theta_A\rangle$ and $\langle(s^2, t^2, u^2 v^2, 2); \mu_B, \theta_B\rangle$ be two Single Valued Pentagonal Neutrosophic Numbers [9].

2.17.1 | Addition

$$A + B = \langle\left((s^1 + s^2), (t^1 + t^2), (u^3 + u^3\right), (v^1 + v^2), (w^1 + w^2), \mu_A \wedge \mu_B, \theta_A \vee \theta_B\rangle.$$

2.17.2 | Subtraction

$$A - B = \langle\left((s^1 - s^2), (t^1 - t^2), (u^3 - u^3\right), (v^1 - v^2), (w^1 - w^2)), \mu_A \wedge \mu_B, \theta_A \vee \theta_B\rangle.$$

2.17.3 | Multiplication

$$A \times B = \langle ((s^1 \times s^2), (t^1 \times t^2), (u^3 \times u^3), (v^1 \times v^2), (w^1 \times w^2)), \mu_A \wedge \mu_B, \theta_A \vee \theta_B \rangle.$$

2.17.4 | Division

$$A \div B = \langle ((s^1 \div s^2), (t^1 \div t^2), (u^3 \div u^3), (v^1 \div v^2), (w^1 \div w^2)), \mu_A \wedge \mu_B, \theta_A \vee \theta_B \rangle.$$

2.17.5 | Scalar multiplication

Let k be the scalar

$$k \times A = \langle ((k \times s^1), (k \times t^1), (k \times u^1), (k \times v^1), (k \times w^1)), \mu_A, \theta_A \rangle,$$

where:

I. μ_A and μ_B are the membership values of A and B, respectively.

II. θ_A and θ_B are the non-membership values of A and B, respectively.

III. $\wedge$ represents the minimum operator.

IV. $\vee$ represents the minimum operator.

2.18 | DeNeutrosophication

2.18.1 | De-Neutrosophication of triangular single-valued neutrosophic number

De-Neutrosophication of Triangular Single-Valued Neutrosophic Number $(\tilde{S}) = \langle (m_1, m_2, m_3 : \mu), (n_1, n_2, n_3 : \upsilon), (p_1, p_2, p_3 : \zeta) \rangle$. The de-Neutrosophic form of $\tilde{S}$ is given by $\widetilde{S_D} = \frac{1}{12}(m_1 + 2m_2 + m_3 + n_1 + 2n_2 + n_3 + p_1 + 2p_2 + p_3)$ [4].

2.18.2 | DeNeutrosophication of a single-valued Neutrosophic number

The SVNN of the form B = (T, I, F) is converted to a crisp value using [1]:

$$D(T, I, F) = \frac{2 + T - I - F}{3}.$$

2.18.3 | DeNeutrosophication of interval-valued Neutrosophic number

The Neutrosophication Function For An Intervel-Valued Neutrosophic number is given in equation $\mathfrak{D}(\widetilde{\ddot{x}}) = \left(\frac{({T_j}^L + {T_j}^U)}{2} + \left(1 - \frac{{I_j}^L + {I_j}^U}{2}\right) * ({I_j}^U) - \left(\frac{{F_j}^L + {F_j}^U}{2}\right) * (1 - {F_j}^U\right)$ Where $(\widetilde{\ddot{x}}) = \langle \lfloor {T_j}^L, {T_j}^U \rfloor, \lfloor I^L, {I_j}^U \rfloor, \lfloor I^L, {T_j}^U \rfloor \rangle$ [2].

2.18.4 | DeNeutrosophication of triangular Neutrosophic number [5]

$$D(\widetilde{A}) = \frac{2 + \left(\frac{a_1 + a_2 + a_3}{3}\right) - \left(\frac{b_1 + b_2 + b_3}{3}\right) - \left(\frac{c_1 + c_2 + c_3}{3}\right)}{3}.$$

2.18.5 | DeNeutrosophication of pentagonal Neutrosophic number

Let a single-valued pentagonal neutrosophic number be represented as $(\tilde{S}s) = \langle [(s^1, t^1, u^1, v^1, w^1); \mu], [(s^2, t^2, u^2, v^2, w^2); \theta], [(s^3, t^3, u^3, v^3, w^3); \eta] \rangle$. The defuzzified value $D(S)$ is given by $D(s) = \frac{1}{6}[s^1 + t^1 + u^1 + v^1 + w^1].\mu + \frac{1}{6}[s^2 + t^2 + u^2 + v^2 + w^2].\theta + \frac{1}{6}[s^3 + t^3 + u^3 + v^3 + w^3].\eta$ [9].

2.18.6 | Defuzzification of generalized trapezoidal neutrosophic number

The defuzzification of Neutrosophic number $\tilde{S}$ with truth membership function $\pi_{\tilde{S}}$, indeterminacy membership function $\theta_{\tilde{S}}$ and falsity membership function $\eta_{\tilde{S}}$ is defined by [7]:

$$R(\tilde{S}) = \frac{1}{3}[R_r(\pi_{\tilde{S}}) + R_r(\theta_{\tilde{S}}) + R_r(\eta_{\tilde{S}})],$$

where R_r is the Rouben's ranking function.

If $\tilde{N} = \langle(m_1, m_2, m_3; \mu), (n_1, n_2, n_3, n_4; \nu), (p_1, p_2, p_3, p_4; \zeta)\rangle$ is a Generalized Trapezoidal Neutrosophic Number (GTNN), then

$$R(\tilde{N}) = \frac{1}{3}\left(\frac{1}{4\mu}\left(2m_2 + (2\mu - 1)(m_1 + m_3)\right) + \frac{1}{4(1-\nu)}\left(2n_2 + (1 - 2\nu)(n_1 + n_3)\right) + \frac{1}{4(1-\zeta)}\left(2p_2 + (1 - 2\zeta)(p_1 + p_3)\right)\right).$$

2.18.7 | De-Neutrosophication of SVTrNN

Suppose $\widetilde{\mathcal{N}}$ be a Neutrosophic number whose parametric form is $\langle[\mathcal{N}_1(\alpha), \mathcal{N}_2(\alpha)], [\mathcal{N}_1'(\beta), \mathcal{N}_2'(\beta)], [[\mathcal{N}_1''(\gamma), \mathcal{N}_2''(\gamma)]\rangle$. Then, the De-Neutrosophication value of $\widetilde{\mathcal{N}}$ is represented by $D(\widetilde{\mathcal{N}})$, and is defined as [8]:

$$De(\widetilde{\mathcal{N}}) = \frac{\alpha\mathcal{N}_1(\alpha) + (1-\alpha)\mathcal{N}_2(\alpha) + \beta\mathcal{N}_1'(\beta) + (1-\beta)\mathcal{N}_2'(\beta) + \gamma\mathcal{N}_1''(\gamma) + (1-\gamma)\mathcal{N}_1''(\gamma)}{3}.$$

where $0 \le \alpha, \beta, \gamma \le 1$.

2.19 | Score and Accuracy Functions of IVTrNeNs

The score function concept is used to find a comparison between two IVTrNeNs. The greater the score function value, the greater the IVTrNeN. According to the base of the score and the accuracy functions of an IVTrNeN $\widetilde{a_N}^{IV}$can be defined as follows [2]:

$$S(\widetilde{a_{N_1}}^{IV}) = \frac{1}{12}\left((8 + (a_1 + b_1 + c_1 + d_1) - (e_1 + f_1 + g_1 + h_1) - (l_1 + m_1 + n_1 + p_1)\right) \times (2 + u_{\widetilde{a_{N_1}}^{IV}}{}^L + u_{\widetilde{a_{N_2}}^{IV}}{}^U - v_{\widetilde{a_{N_1}}^{IV}}{}^L - v_{\widetilde{a_{N_1}}^{IV}}{}^U - w_{\widetilde{a_{N_1}}^{IV}}{}^L - w_{\widetilde{a_{N_1}}^{IV}}{}^U\ S(\widetilde{a_{N_1}}^{IV}) \in [0,1].$$

The accuracy function $A(\widetilde{a_{N_1}}^{IV}) = \frac{1}{4}(a_1 + b_1 + c_1 + d_1 - l_1 - m_1 - n_1 - p_1) \times \left(2 + u_{\widetilde{a_{N_1}}^{IV}}{}^L + u_{\widetilde{a_{N_2}}^{IV}}{}^U - v_{\widetilde{a_{N_1}}^{IV}}{}^L - v_{\widetilde{a_{N_1}}^{IV}}{}^U - w_{\widetilde{a_{N_1}}^{IV}}{}^L - w_{\widetilde{a_{N_1}}^{IV}}{}^U\right)$.

2.19.1 | Score of Neutrosophic set

The score of a Neutrosophic set (or an SVNN) is a measure used to convert a Neutrosophic number into a crisp (real) value to facilitate comparison and ranking among different Neutrosophic numbers. It reflects the overall preference or desirability of the Neutrosophic value based on its truth (T), indeterminacy (I), and falsity (F) membership degrees [1].

Let an SVNN be represented as

$$A = (T, I, F),$$

where:

I. $T \in [0,1]$ represents the truth-membership degree.

II. $I \in [0,1]$ represents the indeterminacy-membership degree.

III. $F \in [0,1]$ represents the falsity-membership degree.

Then, the Score Function of A is defined as

$$S(A) = \frac{1 + T - I - F}{2}.$$

3 | State of the Art of Neutrosophic Sustainable Inventory Management Models

In this section, the existing Neutrosophic-based sustainable inventory models are discussed under three core dimensions.

3.1 | Defect and Deteriorating-Based Neutrosophic Sustainable Inventory Management Models

Defect and imperfect handling are two of the strategies for promoting environmental sustainability. Bhavani et al. [4] framed a Neutrosophic based inventor model considering deterioration and discount on defective items. The triangular Neutrosophic representations, together with Particle Swarm Optimization (PSO), determine the optimal inventory level and costs. Bhavani and Mahapatra [7] designed an inventory model with generalized triangular Neutrosophic cost patterns considering deterioration of the items. The defective and non-defective items are classified, and the faulty items are reworked with demand as the function of quality.

Mohanta et al. [10] developed a Neutrosophic logic integrated inventory model with a two-level partial credit policy for perishable items. The proposed quality promotion and profit maximization model includes the cost parameters of preservation technology. Case studies and comparative analysis are performed to demonstrate the efficiency of Neutrosophic models. Miriam et al. [11] discussed the aspect of the rework warehouse model under a Neutrosophic environment, considering the parameters of quality conservation. The PSO technique is applied in handling quality constraints associated with the reworking of defective items. Washima et al. [12] framed a Neutrosophic inventory model for dealing with imperfect items in a two-layer supply chain system. In this work, the price and stock-dependent demand with uncertainty is modelled using Neutrosophic numbers. The proposed Neutrosophic integrated model is sensitively analyzed to demonstrate the efficiency of Neutrosophic numbers in optimizing the production costs in comparison with fuzzy and crisp representations. Supakar et al. [13] designed a Neutrosophic-based trade credit Economic Order Quantity (EOQ) inventory model for deteriorating items using various variants of PSO techniques. Linear trapezoidal Neutrosophic fuzzy numbers are used in modelling, and optimal solutions are derived using Weighted Quantum-Behaved Particle Swarm Optimization (WQPSO), Gaussian Quantum-Behaved Particle Swarm Optimization (GQPSO), and Adaptive Quantum-Behaved Particle Swarm Optimization (AQPSO). Haque et al. [8] formulated a generalized Neutrosophic inventory model with price and deterioration-dependent demand. The non-linear and intricate parameters are modelled using Laplace transforms.

Rahaman et al. [14] formulated a demand-driven inventory model with trapezoidal Neutrosophic numbers representing the deterioration and pertinent preservation technology parameters. Priskilla et al. [15] developed a manufacturing model considering defective product distribution under a Neutrosophic environment, considering the challenges of environmental degradation, resource wastage, and financial losses. An inventory model with generalized triangular Neutrosophic representations is formulated. This model includes the parameters pertaining to screening mechanisms, supplier discounts, preservation technology, advertising, shortages, and back orders. The comparative analysis of this Neutrosophic model with fuzzy and crisp representations demonstrates the competency of neutrosophy in handling multi-dimensional parameters. Surya et al. [16] formulated a Neutrosophic inventory model with price-dependent demand catering for decaying items. The trapezoidal fuzzy numbers are used in mathematical modelling of the demand parameters. Gudeta et al. [17] framed an inventory model considering a single perishable item contributing to both the economy and organizational development. The uncertainty and imprecision in fixing the price discount of defective items by both the supplier and the retailer are well handled with single-valued TNNs. The neutrosophic fuzzy optimization technique is employed to determine the maximum inventory level of the retailers. Talor's series approximation, Mathematical, and LINGO software are used to determine optimal solutions to the Neutrosophic inventory model. The review of the above works synthesizes the efficacy of Neutrosophic integrated models in dealing with faulty items.

Rahaman et al. [18] developed an EPQ model using a Neutrosophic differential equation approach, where production rate depends on time-impacted demand and stock levels, effectively capturing uncertainty through Neutrosophic calculus and demonstrating superior cost optimization compared to traditional and crisp models. Kausar et al. [19] proposed a Neutrosophic fuzzy inventory management model using pentagonal fuzzy numbers and Graded Mean Integration Representation (GMIR) to handle uncertainty in demand, order quantity, and costs, optimizing total cost under fuzzy conditions and demonstrating robust and adaptable inventory control through case-based analysis. Alioğulları et al. [20] applied the interval-valued neutrosophic fuzzy EDAS method to evaluate and rank sustainability risk mitigation strategies in the automotive supply chain, identifying optimal approaches—such as implementing legal compliance programs—while introducing new sustainability risk dimensions and contributing a novel fuzzy EDAS application to the literature. Dubey and Kumar [21] developed a Neutrosophic inventory model using trapezoidal Neutrosophic numbers to optimize total cost without shortages, providing computational results and comparative evaluations to demonstrate the model's effectiveness in handling both conventional and novel inventory problems. Arshi et al. [22] proposed a nonlinear programming model for multiple attribute group decision-making under interval-valued Neutrosophic numbers, using linguistic variables to capture uncertainty and cognitive hesitancy, enabling decision-makers to identify optimal alternatives even when attributes and preferences are interdependent. Dubey et al. [23] provided a comprehensive survey on applying Neutrosophic principles to inventory management, highlighting how Neutrosophic approaches address uncertain demand and imprecise data more effectively than traditional and fuzzy models, thereby improving decision-making in inventory control systems. Sindhuja et al. [24] proposed a Neutrosophic fuzzy trapezoidal number approach to inventory management that integrates environmental sustainability, optimizing energy consumption, and handling shortages while addressing uncertainties, cost variability, and regulatory challenges to promote resilient and eco-efficient supply chain operations. Das and Islam [25] developed a deterministic multi-item inventory model with quadratic demand, time-dependent holding costs, and deterioration, employing Neutrosophic and Pythagorean Hesitant Fuzzy Programming (PHFP) approaches to handle uncertain hexagonal fuzzy cost parameters, demonstrating improved total cost optimization and robustness over traditional inventory models. Pattnaik et al. [26] developed a Neutrosophic fuzzy inventory control model for pandemic conditions, integrating overage management with profit maximization, and demonstrated through numerical examples and sensitivity analysis how their approach optimizes inventory decisions under COVID-19 disruptions. Deb and Islam [27] developed a Neutrosophic interval-valued goal programming approach for a supply chain inventory model of deteriorating items with time-dependent demand, determining the optimal replenishment strategy to minimize total cost under uncertainty. Mohanta et al. [28] propose a Neutrosophic EOQ model for smart manufacturing that optimizes inventory costs by considering idle times, uncertain parameters, and deriving optimal order quantities through De-Neutrosophication and sensitivity analysis. Rajeswari et al. [29] developed a two-warehouse EOQ model for imperfect items using triangular fuzzy Neutrosophic numbers to handle uncertainty in defective proportions, computing optimal order quantity and profit while performing sensitivity analysis and De-Neutrosophication for practical insights. Surya and Mullai [30] developed a Neutrosophic multi-item inventory control model incorporating budgetary, space, and order constraints, where key parameters are represented as TNNs to more realistically handle uncertainties in determining optimal order quantities. Rajeswari et al. [31] applied Octagonal Fuzzy Neutrosophic Numbers (OFNN) to model a reusable container inventory system, accounting for container shrinkage and uncertain parameters, and demonstrated how De-Neutrosophication via the removal area method provides approximate optimal solutions while highlighting the effect of return rates through sensitivity analysis. Islam and Das [32] developed a multi-objective inventory model for multi-item systems considering time-dependent demand, holding costs, and time-varying deterioration, where shortages are partially backlogged; to reduce deterioration, a preservation condition was incorporated, and uncertainties in cost parameters were modeled using generalized trapezoidal fuzzy numbers, with solutions obtained through Neutrosophic hesitant fuzzy programming, fuzzy nonlinear programming, and fuzzy additive goal programming, validated by numerical examples and sensitivity analysis. Pal and Chakraborty [33] developed a triangular Neutrosophic EPQ model for deteriorating items with ramp-type demand and time discounting, showing that the Neutrosophic approach yields superior and more reliable results than the crisp model. Das and Islam [34] demonstrated the efficiency of their multi-item inventory model, which utilizes a Neutrosophic hesitant fuzzy programming approach to handle uncertain cost parameters under space constraints, through comprehensive numerical and sensitivity analyses.

Table 1. Comparative overview of Neutrosophic-based inventory models for defective and deteriorating items and their optimization approaches.

Author(s)	Neutrosophic Representation	Optimization / Computational Technique	Key Features	Primary Objective
Bhavani et al. [4]	TNNs	PSO	Deterioration, discount, defect	Optimal inventory level and cost
Bhavani and Mahapatra [7]	Generalized TNNs	Analytical optimization	Defective and non-defective classification	Quality-based demand modeling
Mohanta et al. [10]	Neutrosophic logic-based model	Case studies and comparative analysis	Partial credit, perishable goods, preservation technology	Profit maximization, quality promotion
Bhavani and Mahapatra [7]	Neutrosophic environment	PSO	Quality conservation, rework warehouse	Minimize cost and defects
Washima et al. [12]	Trapezoidal Neutrosophic numbers	Sensitivity analysis	Two-layer supply chain, imperfect items	Cost minimization under uncertainty
Supakar et al. [13]	Linear trapezoidal Neutrosophic fuzzy numbers	WQPSO, GQPSO, AQPSO	Trade credit, deterioration	Derive optimal EOQ
Haque et al. [8]	Generalized Neutrosophic numbers	Laplace transform methods	Price and deterioration-dependent demand	Solve non-linear dynamics
Rahaman et al. [14]	Trapezoidal Neutrosophic numbers	Simulation	Deterioration, preservation tech	Demand-driven optimization
Priskilla et al. [15]	Generalized TNNs	Neutrosophic fuzzy optimization, LINGO, Mathematica	Defect distribution, screening, discounts, backorders	Sustainability and profit
Surya et al. [16]	Trapezoidal fuzzy numbers	Analytical comparison	Price-dependent decaying items	Cost minimization
Gudeta et al. [17]	Single-valued TNNs	Neutrosophic fuzzy optimization	Price discount uncertainty, perishables	Maximize retailer inventory level
Rahaman et al. [18]	TNNs	Neutrosophic differential equation	Time-impacted demand, deterioration	Cost minimization
Kausar et al. [19]	Pentagonal fuzzy numbers in Neutrosophic context	GMIR	Fuzzy consumption, cost components	Cost minimization
Alioğulları et al. [20]	Interval-valued Neutrosophic fuzzy numbers	IVN-Fuzzy EDAS (MCDM)	Sustainability risks, automotive supply chain	Strategy ranking
Arshi et al. [22]	Interval-valued Neutrosophic numbers	Nonlinear programming	MADM, aggregation operators	Decision optimization
Sindhuja et al. [24]	Neutrosophic fuzzy trapezoidal numbers	Python-based analysis	Shortages, energy cost, and environmental metrics	Energy optimization
Das and Islam [25]	Hexagonal fuzzy and Neutrosophic hybrid	NHFP and PHFPA	Multi-item quadratic demand	Total cost reduction

Table 1. Continued.

Author(s)	Neutrosophic Representation	Optimization / Computational Technique	Key Features	Primary Objective
Pattnaik et al. [26]	Single-valued triangular Neutrosophic fuzzy numbers	Signed distance method	Overage, pandemic disruption	Profit maximization
Rajeswari et al. [29]	Triangular fuzzy Neutrosophic numbers	De-Neutrosophication via expected value	Two-warehouse EOQ, imperfect items	Profit maximization
Surya and Mullai [30]	TNNs	Lagrange multiplier	Budget, space, demand	Optimal order quantity
Das and Islam [25]	Generalized trapezoidal fuzzy numbers in Neutrosophic form	Neutrosophic hesitant fuzzy programming and fuzzy non-linear programming	Multi-item inventory, demand-dependent production cost, deterioration, space constraint	Multi-objective cost optimization
Das and Islam [34]	TNNs	Analytical optimization	Ramp-type demand, Weibull deterioration, reliability, time discounting, partial backordering	Cost-effective and reliable production planning
Islam and Das [32]	Generalized trapezoidal fuzzy numbers in the Neutrosophic context	Neutrosophic hesitant fuzzy programming, fuzzy additive goal programming	Time-dependent demand and holding cost, deterioration, and preservation condition	Multi-objective inventory optimization
Rajeswari et al. [31]	OFNN	Removal Area method (De-Neutrosophication)	Reusable container inventory, shrinkage, return rate	Optimal container allocation and cost minimization
Mohanta et al. [28]	Single-valued GTNNs	Analytical optimization and mean interval De-Neutrosophication	Idle time, deterioration, smart manufacturing, off-days	Cost minimization
Deb and Islam [27]	Neutrosophic triangular numbers/interval-valued parameters	Neutrosophic Interval Valued Goal Programming (NIVGP)	Deteriorating items, time-dependent demand, trade credit	EOQ optimization, cost minimization
Dubey et al. [23]	Trapezoidal Neutrosophic numbers	Analytical comparison and optimization	Holding, ordering, and shortage costs	EOQ-based cost minimization
Dubey and Kumar [35]	Trapezoidal Neutrosophic numbers	Analytical Neutrosophic optimization	Holding and shortage costs	EOQ inventory model
Dubey et al. [23]	SVNN	Analytical and comparative approach	Demand, holding cost, ordering cost, uncertainty in parameters	To design a cost-effective inventory model incorporating Neutrosophic uncertainty for better decision-making

3.2 | Green Neutrosophic Inventory Models

Researchers have formulated inventory models considering green cost parameters. The costs associated with carbon emissions, including green technology, are included in the Neutrosophic models. The green costings are also discussed with other inventory management situations. Barman et al. [36] proposed a faulty handling multi-objective supply chain[1] inventory model with single-valued trapezoidal Neutrosophic numbers. The implementation of three carbon policies in inventory management, such as carbon tax[2], carbon cap-and-trade, and carbon cap-and-offset are discussed in brief. Sarkar et al. [37] framed a multi-objective Neutrosophic inventory model considering the costs associated with pollution mitigation. The Neutrosophic goal programming technique is applied to minimize the costs and maximize the profit, considering warehouse management and eco-conservation. Kar et al. [38] formulated an inventory model with Neutrosophic demand considering price, marketing, service, and eco parameters. The pentagonal Neutrosophic numbers are used,

[1] https://www.sciencedirect.com/topics/social-sciences/supply-chain-management
[2] https://www.sciencedirect.com/topics/social-sciences/carbon-t ax

and the geometric programming approach is applied in handling the non-linearity of the parameters. Numerical illustrations and comparative analysis demonstrate the efficiency of the proposed Neutrosophic model over fuzzy and crisp representations.

Resource efficiency also amounts to sustainability. In this context, Garg et al. [39] developed an inventory model based on container management organization, focusing on resource efficiency. The imbalance of the flow of containers is well modelled using trapezoidal bipolar Neutrosophic numbers. The algorithmic approach presented in this work predicts the expected inventory costs under various conditions. Sugapriya et al. [40] proposed a container inventory model with a trapezoidal bipolar Neutrosophic number. Comparison with triangular bipolar Neutrosophic numbers demonstrates the efficiency of trapezoidal representations over triangular forms. Mohanta et al. [41] developed a Neutrosophic-based inventory model using generalized trapezoidal single-valued Neutrosophic numbers, considering deteriorating items. The optimal ordering quantity, optimal opening, and closing time are determined using Neutrosophic modelling. Kar et al. [42] contributed a multi-objective perishable multi-item green inventory model considering eco-constraints with Neutrosophic representations. The model addresses the tradeoff between profit and carbon emissions with the objective of profit maximization and emission mitigation. Supakar et al. [43] proposed a green production inventory with Neutrosophic representations to handle uncertain parameters. The artificial bee colony algorithm is used in modeling the green inventory model. Loganayaki et al. [44] discussed a Neutrosophic-based inventory model with TNNs with power demand and preservation technology. The primary objective of this Neutrosophic model is cost minimization, considering shortages and linear holding costs. Sindhuja et al. [24] discussed the strategies of energizing inventory management in handling shortages using Neutrosophic trapezoidal numbers. The need of developing such models in underscoring the balance between economic efficiency and ecological responsibility is well substantiated in the illustration. Saeed et al. [45] discussed green supply chain model with refined four valued Neutrosophic optimization. The objective of this proposed Neutrosophic model is resilience maximization and minimization of Greenhouse Gases (GHG). Pal et al. [46] discussed Neutrosophic fuzzy inventory system with different demand patterns considering the impacts if carbon emissions, inflation and delayed deterioration. Non-linear triangular dense Neutrosophic numbers are used in modelling the uncertain parameters. The environmental metrics are integrated with Neutrosophic inventory models to characterize the fluctuations in sustainable cost parameters.

Mukherjee et al. [47] developed a multi-objective sustainable inventory model incorporating shortages, emissions, Radio Frequency Identification (RFID), and fuzzy-random uncertainties, where demand depends on price and marketing efforts, and demonstrated through sensitivity analysis that their approach outperforms traditional models by providing more reliable and practical decision support under real-world uncertainty. Palanivel and Venkadesh [49] proposed an integrated sustainable production inventory model that incorporates green technologies—both exponential and linear—to minimize carbon emissions, optimize resource use, and enhance supply chain resilience, offering a strategic framework that balances economic efficiency with environmental sustainability through analytical modeling and sensitivity analysis. Nafei et al. [48] proposed a sustainable inventory model with a cap-and-trade policy, incorporating price-dependent demand and inflation effects, to determine optimal inventory levels that balance economic performance with environmental responsibility, validated through numerical illustration and sensitivity analysis.

Table 2. Summary of green Neutrosophic inventory models incorporating environmental costs, sustainability considerations, and optimization techniques.

Researcher(s)	Neutrosophic Representation	Optimization / Method Applied	Focus Area	Environmental or Sustainability Aspect
Barman et al. [36]	Single-Valued Trapezoidal	Multi-objective supply chain model	Faulty handling, supply chain	Carbon tax, cap-and-trade, and cap-and-offset policies integrated
Sarkar et al. [37]	Triangular / Generalized	Neutrosophic goal programming	Multi-objective inventory	Pollution mitigation, eco-conservation
Kar et al. (model 1) [38]	Pentagonal	Geometric programming	Demand-dependent inventory	Price, marketing, service, and eco parameters
Garg et al. [39]	Trapezoidal bipolar	Algorithmic simulation	Container inventory	Resource efficiency and flow balance
Sugapriya et al. [40]	Trapezoidal bipolar vs. triangular	Comparative modelling	Container inventory	Resource utilization
Mohanta et al. [28]	Generalized trapezoidal	Analytical optimization	Deteriorating items	Sustainable ordering strategy
Kar et al. (model 2) [42]	Neutrosophic (general)	Multi-objective programming	Perishable multi-item green inventory	Profit-emission trade-off
Supakar et al. [43]	Neutrosophic	Artificial bee colony algorithm	Green production inventory	Green manufacturing
Loganayaki et al. [44]	Triangular	Analytical modelling	Power demand and preservation tech	Shortages and holding costs
Sindhuja et al. [24]	Trapezoidal	Analytical modelling	Shortage management	Economic–ecological balance
Saeed et al. [45]	Refined four-valued	Neutrosophic optimization	Green supply chain	GHG minimization, resilience maximization
Pal et al. [46]	Non-linear triangular dense	Analytical modelling	Fuzzy–Neutrosophic hybrid system	Carbon emission, inflation, and delayed deterioration
Mukherjee et al. [47]	Fuzzy-random and Neutrosophic environment	Multi-objective optimization with sensitivity analysis	Shortages, RFID, inflation, emission	Profit and cost optimization
Palanivel and Venkadesh [49]	Sustainable production inventory with green technology	Sensitivity analysis and comparative approach	Ramp-type demand, Weibull deterioration, green tech	Carbon emission minimization
Nafei et al. [48]	Neutrosophic numbers	Neutrosophic Autocratic MCDM	Price-dependent demand, carbon cap, inflation	Sustainable inventory control

3.3 | Methodological Approaches of Neutrosophic Sustainable Inventory Management Models

The Neutrosophic-based sustainable inventory models are computationally intensive. The three-valued logic integrated with inventory modelling and sustainability factors increases the complexity of computations. Hence, it is not easy to obtain exact analytical solutions. This is the reason for employing different optimization techniques ranging from classical mathematical programming to advanced and hybrid techniques. Rahaman et al. [18] applied Neutrosophic differential equations in modelling the Economic Production Quantity (EPQ) inventory model with production rate as a function of demand and stock. Momena et al. [50] used first-order linear Neutrosophic differential equations for handling an inventory system with insufficient data. The exploration of differential equations and numerical simulation facilitates in alleviating uncertainty in inventory systems. Haque et al. [51] applied fractional calculus for Neutrosophic -valued functions in inventory lot sizing. The choice of fractional calculus is made to capture the system's memory and past experiences, paving the way for enhanced decision-making. Kumar and Singh [52] applied the PSO algorithm in optimizing the Neutrosophic inventory model with quality-dependent demand and payment delay. Kalaiarasi et al. [53] introduced a Neutrosophic integrated vendor -buyer model with different parameters. PSO technique, MATLAB, and Python are applied for simulation and sensitivity analysis. GJ et al. [54] used Genetic Algorithms (GAs) to optimize the Neutrosophic EOQ model with varying patterns of demand and lead time. Bhavani et al. [4] used the PSO method to devise a solution to the Neutrosophic inventory model addressing deterioration and discounts.

Kalaiarasi et al. [53] discussed inventory modelling using Neutrosophic logic. The benefits of Neutrosophic modelling in handling higher ranges of uncertainty are outlined with suitable illustrations. Bolos et al. [55] employed Neutrosophic-based nonlinear mathematical programming algorithms in framing production stock management-based inventory models. Deb and Islam [27] applied the Neutrosophic goal programming technique with the representations of triangular numbers. This model deals with the aspects of deterioration and credit risk in the supply chain. Kalaiarasi and Swathi [56] discussed the situation of quick returns. The study explored two different types of Neutrosophic numbers in modelling perfect rate, demand rate, purchase costs, and ideal order quantity. Mullai and Surya [57] discoursed on backorders to handle the shortage using Neutrosophic representations. Das and Islam [58] developed a multi-objective shortage inventory model and determined a solution using Neutrosophic non-linear programming. Different programming methods are applied under the environments of fuzzy and intuitionistic, and it is observed that Neutrosophic-based programming methods are more effective in handling complexity. Jayanthi [59] discussed overage management using Neutrosophic fuzzy geometric programming. The computational complexity of the Neutrosophic inventory models is well handled with different programming techniques.

Miriam et al. [60] proposed a Neutrosophic inventory model integrated with coordinated rework stations and distribution centers, optimized using Ant Colony Optimization (ACO), to enhance supply chain efficiency, minimize costs, and strengthen resilience against uncertainties through the combined use of Neutrosophic logic and swarm intelligence. Grützner et al. [61] developed an interlinked process-reference model for mature inventory management that supports supply chain automation through a comprehensive multi-criteria ABC analysis, integrating empirical insights from case studies, surveys, and literature to promote resilient, value-added, and sustainable inventory practices for targeted automation and decision-making. Dwivedi et al. [62] proposed a dynamic pharmaceutical inventory model for pandemic conditions that integrates environmental emission rates, preservation technology, and fuzzy learning, using advanced optimization algorithms like Ant Colony and Cuckoo Search to optimize pricing, investment, and replenishment strategies, thereby balancing profitability, sustainability, and public health objectives under uncertainty. Kar et al. [42] developed multi-objective perishable multi-item green inventory models with stock-dependent demand under uncertain finite time horizons, using a Neutrosophic optimization approach to maximize profit while minimizing wastage and carbon emissions, and demonstrated trade-offs and sensitivity through numerical examples. Anand et al. [63] applied a Neutrosophic transportation problem with a contingency technique to optimize electricity distribution, leveraging Neutrosophic logic to handle uncertainty, indeterminacy, and inconsistency, and using center-of-gravity and score functions to determine crisp values and minimize transportation costs effectively. GJ et al. [54] developed a Neutrosophic EOQ model that incorporates demand and lead-time uncertainties, using GAs and simulated annealing to optimize order quantities, demonstrating effective inventory management in highly uncertain environments through practical examples. Kousar et al. [64] developed a Multi-Objective Neutrosophic Fuzzy Linear Programming (MONFLP) model to optimize Kharif and Rabi crop production under uncertainty, maximizing net profit and output while accounting for climate, water availability, and irrigation constraints, demonstrating the model's effectiveness in handling unpredictable agricultural parameters in Pakistan. Cakmak and Guney [65] proposed a two-stage Neutrosophic fuzzy EDAS framework integrating T2NN-LOPCOW and T2NN-ARAS methods to classify spare parts inventory and evaluate industry 4.0-based material handling technologies, demonstrating through a real-life aviation case study that Automated Guided Vehicles (AGVs) are the most effective solution and highlighting the model's robustness and practical applicability. Adhami et al. [66] developed a Neutrosophic Compromise Programming Approach (NCPA) for multilevel supplier selection under fuzzy supply and demand, using membership, nonmembership, and indeterminacy degrees to handle uncertainty and provide satisfactory hierarchical decision-making solutions, validated through a numerical example. Cakmak and Guney [65] applied the Neutrosophic Fuzzy EDAS method to classify spare parts inventories in the aviation industry, enhancing inventory management efficiency under uncertainty while minimizing costs and ensuring high service levels. Badhotiya et al. [67] propose a Neutrosophic programming approach for integrated production-distribution planning in a two-echelon supply chain, addressing uncertainties and conflicting objectives—minimizing total cost, delivery time, and backorder level—through a tri-objective MILP model validated via a real-world automotive case study and Pareto optimality analysis. Daham [68] addresses discrete facility location problems under uncertainty by formulating Neutrosophic models that handle vague and ambiguous data for locations, distances, times, and costs, providing numerical examples to demonstrate the efficacy of these models compared to traditional deterministic approaches. Jdid et al. [69] employed Neutrosophic logic to enhance the static inventory model with deficit, allowing for more accurate handling of uncertain demand fluctuations and optimizing inventory levels while balancing deficit and storage costs. Mondal et al. [70] developed a Neutrosophic optimization-based EOQ model for seasonal products with logistic-growth demand, Weibull-distributed deterioration, partial backordering, and fully permissible delay in payment, effectively handling imprecise real-world information and providing managerial insights on cost minimization and inventory depletion timing under uncertainty. Haq et al. [71] developed a multi-criteria fuzzy Neutrosophic decision-making model for complex multi-site supply chain networks, integrating truth, indeterminacy, and falsity membership functions to optimize transportation cost and delivery time under uncertainty, and demonstrated its effectiveness through an industrial case study compared to other approaches. Piper [72] developed a Neutrosophic geometric programming-based inventory model that surpasses traditional optimization methods by effectively handling space constraints and uncertain, imprecise parameters. Kar et al. [73] extended the classical static inventory management model without deficit by incorporating Neutrosophic logic, allowing for indeterminate demand rates and uncertain data, which provides a more realistic and safe framework for determining the ideal inventory volume and associated costs, thereby enhancing operational efficiency and profitability. Islam [74] developed a multi-objective inventory model for deteriorating items under storage space constraints, incorporating time-dependent demand and holding costs, partial backlogging, and uncertain parameters modeled as generalized trapezoidal fuzzy numbers; the model is solved using Neutrosophic hesitant fuzzy programming and fuzzy nonlinear programming, with numerical examples and sensitivity analysis validating its applicability. Ahmad et al. [75] introduced a multiobjective framework using a Neutrosophic goal programming approach to address uncertainty in optimal shale gas water management. Their paper presented a Neutrosophic optimization model and algorithm for shale gas water management, which addresses conflicting constraints related to fresh water and wastewater under uncertain conditions. Islam and Kundu [76] developed a Neutrosophic optimization-based inventory model without shortages, applying geometric programming and additive operator methods to manage uncertainty effectively in production and storage parameters.

Table 3. Overview of Neutrosophic-based inventory, supply chain, and optimization models with associated solution techniques and key features.

Author(s)	Method/ Technique Used	Model Type	Key Features
Rahman et al. [18]	Neutrosophic differential equations	EPQ	Production rate as a function of demand and stock
Momena et al. [50]	First-order linear Neutrosophic differential equations	Inventory with insufficient data	Uncertain demand and stock levels

Table 3. Continued.

Author(s)	Method/ Technique Used	Model Type	Key Features
Haque et al. [8]	Fractional calculus for Neutrosophic-valued functions	Inventory lot sizing	Fractional derivative to capture system memory
Kumar and Singh [52]	PSO	Quality-dependent demand with payment delay	Swarm intelligence for parameter tuning
Kalaiarasi et al. [53]	PSO, MATLAB, and Python Simulation	Vendor–buyer integrated model	Multi-parameter analysis
GJ et al. [54]	GA	EOQ with variable demand and lead time	Evolutionary search technique
Bhavani et al. [4]	PSO	Deteriorating items with discounts	Price–time deterioration factor
Moorthy et al. [77]	Neutrosophic logic framework	General inventory modelling	Illustrative uncertainty handling
Bolos et al. [55]	Nonlinear neutrosophic mathematical programming	Production–stock management	Nonlinear constraints
Deb and Islam [58]	Neutrosophic goal programming (triangular numbers)	Deterioration and credit risk	Multi-objective framework
Kalaiarasi and Swathi [56]	Neutrosophic numbers (two types)	Quick returns scenario	Purchase cost, demand rate, ideal order quantity
Mullai and Surya [57]	Neutrosophic representation for backorders	Shortage management	Backorder and shortage rate
Das and Islam [58]	Neutrosophic nonlinear programming	Multi-objective shortage inventory	Cost and shortage objectives
Jayanthi [59]	Neutrosophic fuzzy geometric programming	Overage management	Cost and volume optimization
Miriam et al. [60]	Neutrosophic inventory model	ACO	Optimize the supply chain, rework, and cost
Grützner et al. [61]	Multi-criteria Neutrosophic evaluation	Process-reference modeling	MCABC analysis, automation, sustainability
Dwivedi et al. [62]	Fuzzy learning–based Neutrosophic hybrid	Ant Colony and Cuckoo Search	Price, infection rate, and environmental costs
Kar et al. [42]	Neutrosophic optimization	Weighted-sum and GRG (LINGO)	Perishability, carbon emission, and wastage
Anand et al. [63]	Neutrosophic fuzzy numbers	Contingency means allocation	Electricity distribution, cost uncertainty
GJ et al. [54]	Neutrosophic EOQ framework	GA and Simulated Annealing	Demand, lead time, and inventory cost
Cakmak and Guney [65]	Neutrosophic Fuzzy EDAS	MCDM approach	Aviation spare parts classification
Badhotiya et al. [67]	Neutrosophic membership functions	Tri-objective mixed integer programming	Cost, delivery, backorder
Daham [68]	Neutrosophic parameters	Mathematical modeling	Facility location, cost, distance
Haq et al. [71]	Neutrosophic set functions	Compromise programming	Multi-objective supply chain
Mondal et al. [70]	Neutrosophic coefficients	Weighted arithmetic mean	Logistic demand, delay, deterioration
Haq et al. [71]	Neutrosophic set functions	Compromise programming	Multi-objective supply chain
Kar et al. [73]	Neutrosophic set functions	Geometric programming	Setup cost, production, storage
Islam and Kundu [76]	Neutrosophic geometric programming and additive operator	Inventory model without shortages	Time-dependent holding cost, inverse relation of setup and production cost, storage imprecision
Jdid et al. [78]	Neutrosophic logic modeling	Static inventory without a deficit	Demand indeterminacy, ideal inventory volume

Table 3. Continued.

Author(s)	Method/ Technique Used	Model Type	Key Features
Jdid et al. [78]	Neutrosophic logic-based mathematical model	Static inventory with a deficit	Deficit cost, demand fluctuation
Cakmak and Guney [65]	Neutrosophic Fuzzy EDAS (MCDM)	Aviation spare parts inventory	Multi-criteria classification, uncertain demand
Adhami et al. [66]	NCPA	Supplier selection decision model	Multi-level decision structure, fuzzy supply and demand
Kousar et al. [64]	MONFLP	Crop production optimization	Canal irrigation, crop yield, and environmental limits
Ahmad et al. [75]	Neutrosophic goal programming approach	Multi-objective optimization model for shale gas water management	Freshwater allocation, wastewater treatment, environmental and socio-economic trade-offs, and uncertain parameters
Islam [74]	Neutrosophic hesitant fuzzy programming and fuzzy non-linear programming	Multi-objective inventory model for deteriorating items under space constraint	Time-dependent demand and holding cost, partial backlogging, generalized trapezoidal fuzzy parameters

4 | Future Directions

The Neutrosophic-based sustainability inventory models shall be extended to handle the challenges of Industry 5.0. The integration of artificial intelligence, blockchain, and the Internet of Things enhances production. However, the amalgamation of these aspects incurs additional costs, which require Neutrosophic representations. The development of such Neutrosophic models facilitates real-time decision making. In addition to the existing optimization algorithms, a few bio-inspired metaheuristic algorithms shall be developed to obtain better solution accuracy. The sustainability-focused Neutrosophic models shall be further explored with the association of carbon costings. Multidimensional inventory models addressing multiple aspects of inventory handling shall be developed. Supply chain inventory models with Neutrosophic representations shall also be formulated. The researchers shall also make a comparative analysis of the Neutrosophic inventory models to demonstrate the robustness and efficacy of Neutrosophic representations.

5 | Conclusion

The review of Neutrosophic sustainability inventory models presented in this study exhibits a comprehensive study of Neutrosophic integrated inventory models. The characterization of NS and Neutrosophic numbers is presented in detail. Neutrosophic inventory models considering sustainability are discussed under three aspects: defective models, green models, and methodological approaches. The existing Neutrosophic-based SIM contributes to the literature of inventory modelling and occupies a significant position. Neutrosophic-based inventory models facilitate enhanced decision-making. The models integrated with diverse optimization algorithms also support deriving optimal solutions with more precision and accuracy. As the industries are advancing with smart and sustainable technologies, they face several intricate challenges. This could be well handled with Neutrosophic inventory modelling.

Authors' Contributions

All aspects of the research and manuscript preparation were carried out by the author. The author has read and approved the final version of the manuscript.

Funding

This study did not receive any specific funding from public, commercial, or non-profit funding agencies.

Data Availability

All data are included in the text.

Conflict of Interest

The author declares that he does not have any conflict of interest.

Consent for Publication

The author has given consent for the publication of this manuscript.

Ethics Approval and Consent to Participate

This study does not involve any research conducted on human participants or animals.

References

[1] Kahraman, C., Oztaysi, B., & Cevik Onar, S. (2020). Retracted: Single & interval-valued Neutrosophic AHP methods: Performance analysis of outsourcing law firms. *Journal of intelligent & fuzzy systems*, *38*(1), 749–759. https://doi.org/10.3233/JIFS-179446

[2] Saini, R. K., Sangal, A., & Ahirwar, A. (2022). A novel approach by using interval-valued trapezoidal Neutrosophic numbers in transportation problem. *Neutrosophic sets syst*, *51*, 234–253. https://fs.unm.edu/nss8/index.php/111/article/view/2561

[3] Wang, Q., Huang, Y., Kong, S., Ma, X., Liu, Y., Das, S. K., & Edalatpanah, S. A. (2021). A novel method for solving multiobjective linear programming problems with triangular Neutrosophic numbers. *Journal of mathematics*, *2021*(1), 6631762. https://doi.org/10.1155/2021/6631762

[4] Bhavani, G. D., Georgise, F. B., Mahapatra, G. S., & Maneckshaw, B. (2022). Neutrosophic cost pattern of inventory system with novel demand incorporating deterioration and discount on defective items using particle swarm algorithm. *Computational intelligence and neuroscience*, *2022*(1), 7683417. https://doi.org/10.1155/2022/7683417

[5] Martin, N., Mayan, M. K., & Smarandache, F. (2020). Neutrosophic optimization of industry 4.0 production inventory model. *Neutrosophic sets and systems*, *38*, 470-481. https://doi.org/10.5281/zenodo.4300572

[6] Negi, A., & Singh, O. (2025). Probabilistic inventory model for deteriorating items with uncertain demand under pentagonal fuzzy environment. *Reliability: Theory & applications*, *20*(1(82)), 754–772. https://doi.org/10.24412/1932-2321-2025-182-754-772

[7] Bhavani, G. D., & Mahapatra, G. S. (2023). Inventory system with generalized triangular Neutrosophic cost pattern incorporating maximum life-time-based deterioration and novel demand through PSO. *Soft computing*, *27*(5), 2385–2402. https://doi.org/10.1007/s00500-022-07769-3

[8] Haque, R., Rahaman, M., Alam, S., Behera, P. K., & Mondal, S. P. (2025). Generalized Neutrosophic Laplace transform and its application in an EOQ model with price and deterioration-dependent demand. *Opsearch*, *62*(2), 550–582. https://doi.org/10.1007/s12597-024-00803-y

[9] Chakraborty, A., Banik, B., Mondal, S. P., & Alam, S. (2020). Arithmetic and geometric operators of pentagonal Neutrosophic number and its application in mobile communication service based MCGDM problem. *Neutrosophic sets and systems*, *32*, 61–79. https://digitalrepository.unm.edu/nss_journal/vol32/iss1/6

[10] Mohanta, K., Jha, A. K., Dey, A., & Pal, A. (2023). An application of Neutrosophic logic on an inventory model with two-level partial trade credit policy for time-dependent perishable products: K. Mohanta et al. *Soft computing*, *27*(8), 4795-4822. https://doi.org/10.1007/s00500-022-07619-2

[11] Miriam, M. R., Martin, N., Aleeswari, A., & Broumi, S. (2023). Rework warehouse inventory model for product distribution with quality conservation in Neutrosophic environment. *International journal of Neutrosophic science (IJNS)*, *21*(2), 150–160. https://doi.org/10.54216/IJNS.210215

[12] WashimaKhatun, M., Pal, S., Chakraborty, A., & Shaw, A. K. (2024). A novel technique to detect an optimal two layer supply chain model for imperfect items under various Neutrosophic environment. *International journal of applied and computational mathematics*, *10*(3), 121. https://doi.org/10.1007/s40819-024-01748-y

[13] supakar, P., Kumar, N., Mahato, S. K., & Pal, P. (2024). Neutrosophic trade-credit EOQ model for deteriorating items considering expiration date of the items using different variants of particle swarm optimizations. *International journal of system assurance engineering and management*, *15*(3), 1147–1171. https://doi.org/10.1007/s13198-023-02197-4

[14] Rahaman, M., Haque, R., Salahshour, S., Alam, S., & Prasad, M. S. (2025). Analysis of a demand-driven production inventory model in a trapezoidal Neutrosophic number-ruled decision environment. *Yugoslav journal of operations research*, 35(4), 797-836. https://doi.org/10.2298/YJOR240515052R

[15] Priskilla, B., Baranidharan, B., & Mahapatra, G. S. (2025). Optimizing defective product distribution for sustainability and profitability in manufacturing model under Neutrosophic environment. *Neutrosophic sets and systems*, *86*(1), 28. https://digitalrepository.unm.edu/nss_journal/vol86/iss1/28

[16] Surya, R., Mullai, M., & Vetrivel, G. (2024). Neutrosophic inventory system for decaying items with price dependent demand. *Neutrosophic sets and systems*, *66*(1), 4. https://digitalrepository.unm.edu/nss_journal/vol66/iss1/4

[17] Gudeta, G. A., Thillaigovindan, N., & Wole, G. A. (2025). Inventory control of single perishable item in Neutrosophic environment: With stock-based demand, preservation technology, shortage and two-stage price discount for imperfect items. *Neutrosophic sets and systems*, *77*(1), 11. https://doi.org/10.5281/zenodo.14120586

[18] Rahaman, M., Haque, R., Salahshour, S., Sobczak, A., Azizzadeh, F., Alam, S., & Mondal, S. P. (2025). Solution of an uncertain EPQ model using the Neutrosophic differential equation approach. *Operations research and decisions*, *35*(2), 55–92. http://dx.doi.org/10.37190/ord250203

[19] Kausar, N., Broumi, S., & Cagin, T. (2025). The analysis of pentagonal fuzzy numbers in a Neutrosophic fuzzy inventory management modelling with minimal insufficient supply required and fuzzy consumption. *International journal of Neutrosophic science (IJNS)*, *25*(4), 472-483. https://doi.org/10.54216/IJNS.250440

[20] Alioğulları, E., Türkan, Y. S., Çakmak, E., & Tirkolaee, E. B. (2024). Evaluation of risk strategies for supply chain sustainability with interval-valued Neutrosophic fuzzy EDAS. *Heliyon*, *10*(19), 1-22. https://doi.org/10.1016/j.heliyon.2024.e38607

[21] Dubey, A., & Kumar, R. (2024). Neutrosophic inventory management: A cost-effective approach. *Economics*, *18*(1), 1-15. https://doi.org/10.1515/econ-2022-0101

[22] Arshi, M., Hadi-Vencheh, A., Aazami, A., & Jamshidi, A. (2024). A nonlinear model to solve multiple attribute decision-making problems with interval-valued Neutrosophic numbers. *International journal of industrial engineering,* 35(4), 1-15. (**In Persian**). https://doi.org/10.23055/ijietap.2023.30.3.8023

[23] Dubey, A., Dey, A., Broumi, S., & Kumar, R. (2024). A survey on Neutrosophic principles for inventory management problem. *Neutrosophic sets and systems*, *68*(1), 19. https://digitalrepository.unm.edu/nss_journal/vol68/iss1/19.

[24] Sindhuja, N., Kumari, M. S., Kalaiarasi, K., GJ, M., & HY, S. (2024). Energizing inventory management to optimize energy consumption of handling shortages by Neutrosophic fuzzy trapezoidal number. *International journal of Neutrosophic science (IJNS),* 24(1), 219-236. https://doi.org/10.54216/IJNS.240120

[25] Das, K., & Islam, S. (2024). A deterministic multi-item inventory model with quadratic demand under Neutrosophic and pythagorean hesitant fuzzy programming approach. *Results in control and optimization*, *14*, 100367. https://doi.org/10.1016/j.rico.2023.100367

[26] Pattnaik, S., Nayak, M. M., & Kumar, P. (2023). An inventory control model in the framework of COVID-19 disruptions considering overage items with Neutrosophic fuzzy uncertainty. *Neutrosophic sets and systems*, *56*(1), 9. https://digitalrepository.unm.edu/nss_journal/vol56/iss1/9

[27] Deb, S. C., & Islam, S. (2023). Application of Neutrosophic interval valued goal programming to a supply chain inventory model for deteriorating items with time dependent demand. *Neutrosophic sets and systems*, *53*(1), 35. https://digitalrepository.unm.edu/nss_journal/vol53/iss1/35

[28] Mohanta, K., Sarkar, S., Dey, A., De, D., & Pal, A. (2023). *Cost optimization of an inventory control problem in Neutrosophic environment for smart manufacturing.* https://doi.org/10.21203/rs.3.rs-2590887/v1

[29] Rajeswari, S., Sugapriya, C., Pillai, N. D., Broumi, S., & Smarandache, F. (2022). Application of triangular fuzzy Neutrosophic number to an effective economic order quantity model. In *Handbook of research on advances and applications of fuzzy sets and logic* (pp. 639-665). IGI Global Scientific Publishing. https://doi.org/10.4018/978-1-7998-7979-4.ch029

[30] Surya, R., & Mullai, M. (2022). Neutrosophic multi-item inventory control models with constraints. *Journal of intelligent & fuzzy systems*, *43*(6), 8127–8136. https://doi.org/10.3233/JIFS-221143

[31] Rajeswari, S., Sugapriya, C., Nagarajan, D., Broumi, S., & Smarandache, F. (2021). Octagonal fuzzy Neutrosophic number and its application to reusable container inventory model with container shrinkage. *Computational and applied mathematics*, *40*(8), 308. https://doi.org/10.1007/s40314-021-01600-9

[32] Islam, S., & Das, K. (2021). Multi-objective inventory model with deterioration under space constraint: Neutrosophic hesitant fuzzy programming approach. *Neutrosophic sets and systems,* 47(1), 10. https://digitalrepository.unm.edu/nss_journal/vol47/iss1/10

[33] Pal, S., & Chakraborty, A. (2020). Triangular Neutrosophic based production reliability model of deteriorating item with ramp type demand under shortages and time discounting. *Neutrosophic sets and systems*, *35*(1), 20. https://digitalrepository.unm.edu/nss_journal/vol35/iss1/20

[34] Das, S. K., & Islam, S. (2019). *Fuzzy multi item inventory model with deterioration and demand dependent production cost under space constraint: Neutrosophic hesitant fuzzy programming approach*. Infinite Study. https://doi.org/10.5281/zenodo.3382560

[35] Dubey, A., & Kumar, R. (2024). Enhancing inventory models using fuzzy extended principles for optimization. *Journal of fuzzy extension and applications*, *5*(4), 622–633. https://doi.org/10.22105/jfea.2024.449723.1425

[36] Barman, H., Roy, S. K., Sakalauskas, L., & Weber, G. W. (2023). Inventory model involving reworking of faulty products with three carbon policies under Neutrosophic environment. *Advanced engineering informatics*, *57*, 102081. https://doi.org/10.1016/j.aei.2023.102081

[37] Sarkar, D., & Srivastava, P. K. (2022). A dynamic optimization approach for multi-item multi-objective fixed-place Neutrosophic fuzzy inventory and green management system. *Mathematics in engineering, science & aerospace (MESA), 13*(2). https://logon.ebsco.zone/api/dispatcher/oauth/authorize?client_id=awgycIx57MrwnDQ5h4Ue6

[38] Kar, C., Roy, T. K., & Maiti, M. (2022). EOQ model with price, marketing, service and green dependent Neutrosophic demand under uncertain resource constraint: A geometric programming approach. *Neutrosophic sets and systems*, *51*(1), 49. https://digitalrepository.unm.edu/nss_journal/vol51/iss1/49

[39] Garg, H., Rajeswari, S., Sugapriya, C., & Nagarajan, D. (2022). A model for container inventory with a trapezoidal bipolar Neutrosophic number. *Arabian journal for science and engineering*, *47*(11), 15027–15047. https://doi.org/10.1007/s13369-022-06788-4

[40] Sugapriya, C., Rajeswari, S., Nagarajan, D., Zararsız, Z., & Mahad Al Amri, Z. S. (2022). An effective container inventory model under bipolar Neutrosophic environment. *Neutrosophic sets and systems*, *50*(1), 19. https://digitalrepository.unm.edu/nss_journal/vol50/iss1/19

[41] Mohanta, K., Sarkar, S., Dey, A., De, D., & Pal, A. (2023). *Coptimization of an inventory control problem in Neutrosophic environment for smart manufacturing*. https://doi.org/10.21203/rs.3.rs-2590887/v1

[42] Kar, C., De, M., Maiti, M., & Das, P. (2024). Multi-objective perishable multi-item green inventory models with uncertain finite time horizons and constraints by Neutrosophic optimisation approach. *International journal of mathematics in operational research*, *27*(2), 167–198. https://doi.org/10.1504/IJMOR.2024.137053

[43] Supakar, P., Manna, A. K., Mahato, S. K., & Bhunia, A. K. (2024). Application of artificial bee colony algorithm on a green production inventory problem with preservation for deteriorating items in Neutrosophic fuzzy environment. *International journal of system assurance engineering and management*, *15*(2), 672–686. https://doi.org/10.1007/s13198-022-01692-4

[44] Loganayaki, S., Rajeswari, N., Kirupa, K., & Broumi, S. (2024). Optimization of triangular Neutrosophic based economic order quantity model under preservation technology and power demand. *Neutrosophic*

sets and systems, *65*(1), 11. https://books.google.nl/books?id=nHI_EQAAQBAJ&printsec=frontcover#v=onepage&q&f=false

[45] Saeed, A., Jian, M., Imran, M., Freen, G., & ur Rehman Majid, A. (2024). Green-resilient model for smartphone closed-loop supply chain network design: A novel four-valued refined Neutrosophic optimization. *Computers & industrial engineering*, *190*, 110087.https://doi.org/10.1016/j.cie.2024.110087

[46] Pal, S., Chakraborty, A., & Mahapatra, G. S. (2025). A case study of the impact of carbon emissions and inflation on nonlinear dense Neutrosophic fuzzy inventory system of varying demand with delayed deterioration. *RAIRO-operations research*, *59*(1), 193-218. https://doi.org/10.1051/ro/2024078

[47] Mukherjee, A. K., Maity, G., Jablonsky, J., Roy, S. K., & Weber, G. W. (2025). Multi-objective sustainable inventory model with shortages and radio frequency identification under uncertain environment. *International journal of systems science: Operations & logistics*, *12*(1), 2470917. https://doi.org/10.1080/23302674.2025.2470917

[48] Nafei, A., Huang, C. Y., Chen, S. C., Huo, K. Z., Lin, Y. C., & Nasseri, H. (2023). Neutrosophic autocratic multi-attribute decision-making strategies for building material supplier selection. *Buildings*, *13*(6), 1373. https://doi.org/10.3390/buildings13061373.

[49] Palanivel, M., & Venkadesh, M. (2025). Integrated strategies for sustainable production inventory model with carbon emission reduction with green technology. *Operations research forum, 6*(2), 51. https://doi.org/10.1007/s43069-025-00445-w

[50] Momena, A. F., Haque, R., Rahaman, M., Salahshour, S., & Mondal, S. P. (2024). The existence and uniqueness conditions for solving Neutrosophic differential equations and its consequence on optimal order quantity strategy. *Logistics*, *8*(1), 18. https://doi.org/10.3390/logistics8010018

[51] Haque, R., Rahaman, M., Alrasheedi, A. F., Chalishajar, D., & Mondal, S. P. (2025). Fractional calculus for Neutrosophic-valued functions and its application in an inventory lot-sizing problem. *Fractal and fractional*, *9*(7), 433. https://doi.org/10.3390/fractalfract9070433

[52] Kumar, S., & Singh, S. (2025). Optimization of inventory management for deteriorating items with quality-dependent demand under Neutrosophic environments and payment delay policies. *International journal of scientific research in science, engineering and technology*, *12*, 1395–1408. https://doi.org/10.32628/IJSRSET2512185

[53] Kalaiarasi, K., Anitha, N., Swathi, S., & Ranjitha, B. (2024). Optimization of Neutrosophic vendor-buyer economic order quantity model using particle swarm optimization. *International journal of Neutrosophic science (IJNS), 23*(4), 181-193. https://doi.org/10.54216/IJNS.230414

[54] GJ, M., Anitha, N., Pushpalatha, A. P., Laxmi, K. V., Premalatha, M., & Selvaraj, M. (2024). Optimization of Neutrosophic EOQ model for effective demand management in uncertain environment using genetic optimization. *International journal of Neutrosophic science (IJNS)*, *24*(3), 1-12. https://doi.org/10.54216/IJNS.240305

[55] Boloș, M. I., Sabău-Popa, C. D., & Rus, L. (2023). Optimal management of production stocks with the Neutrosophic fuzzy numbers. *Economic computation & economic cybernetics studies & research,* 57(2), 21-40. https://doi.org/10.24818/18423264/57.2.23.02

[56] Kalaiarasi, K., & Swathi, S. (2023). Neutrosophic inventory model with quick return for damaged materials and python-analysis. In *2023 Neutrosophic superhyperalgebra and new types of topologies* (PP. 219-258). Infinite Study. https://philpapers.org/archive/SMANSA-2.pdf

[57] Mullai, M., & Surya, R. (2020). Neutrosophic inventory backorder problem using triangular Neutrosophic numbers. *Neutrosophic sets and systems*, *31*(1), 11. https://fs.unm.edu/nss8/index.php/111/article/view/438

[58] Das, K., & Islam, S. (2022). A multi-objective shortage follow inventory (SFI) model involving ramp-type demand, time varying holding cost and a marketing cost under Neutrosophic programming approach. *Neutrosophic sets and systems*, 49, 48-69. https://doi.org/10.5281/zenodo.6426371

[59] Jayanthi, K. (2022). A fuzzy differential approach to an overage management with customer acquisition inventory problem via Neutrosophic fuzzy geometric programming technique. *Journal of algebraic statistics, 13*(3). https://logon.ebsco.zone/api/dispatcher/oauth/authorize?client_id=awgycix57mrwndq5h4ue6ycvep0r5mt9&grant

[60] Miriam, M. R., Martin, N., Rezaei, A., & Edalatpanah, S. A. (2025). Neutrosophic inventory model with coordinated rework stations and distribution centers with ant colony optimization. In *Neutrosophic and plithogenic inventory models for applied mathematics* (pp. 247-286). IGI Global Scientific Publishing. https://doi.org/10.4018/979-8-3693-3204-7.ch011

[61] Grützner, L., Voss, D., & Breitner, M. H. (2025). Mature inventory management for supply chain automation: An interlinked process-reference model. *Electronic markets*, *35*(1), 34. https://doi.org/10.1007/s12525-025-00783-x

[62] Dwivedi, V., Keswani, M., Khedlekar, U. K., & Kumar, L. (2025). Optimizing pharmaceutical inventory and investment strategies during pandemics: A dynamic approach integrating environmental emission rates and advanced optimization algorithms. *Operations research and decisions, 3*(2), 23-*35*. http://dx.doi.org/10.37190/ord250202

[63] Anand, M. C. J., Anjana, V. A., Ashwini, R., Varsha, P., Savanur, V. V., & Devana, P. S. (2024). Optimizing electricity distribution through Neutrosophic transportation problem using contingency technique. *2024 IEEE third international conference on power electronics, intelligent control and energy systems (ICPEICES)* (pp. 376-381). IEEE. https://doi.org/10.1109/ICPEICES62430.2024.10719090

[64] Kousar, S., Sangi, M. N., Kausar, N., Pamučar, D., Ozbilge, E., & Cagin, T. (2023). Multi-objective optimization model for uncertain crop production under Neutrosophic fuzzy environment: A case study. *AIMS mathematics, 8*(3), 7584–7605. https://www.aimspress.com/article/doi/10.3934/math.2023380

[65] Cakmak, E., & Guney, E. (2023). Spare parts inventory classification using Neutrosophic fuzzy EDAS method in the aviation industry. *Expert systems with applications*, *224*, 120008. https://doi.org/10.1016/j.eswa.2023.120008

[66] Adhami, A. Y., Melethil, A., & Ahmad, F. (2023). Neutrosophic programming approach to multilevel decision-making model for supplier selection problem in a fuzzy situation. *RAIRO-operations research,* 57(3), 1307-1328. https://doi.org/10.1051/ro/2023064

[67] Badhotiya, G. K., Soni, G., Nepal, B., & Mittal, M. L. (2022). Integrated production-distribution planning optimization using Neutrosophic programming. *International journal of industrial engineering,* 29(2). https://doi.org/10.23055/ijietap.2022.29.2.7497

[68] Daham, H. A. (2022). Neutrosophic discrete facility location problems. *International journal of neutrosophic science (IJNS)*, *19*(1), 30-46. https://doi.org/10.54216/IJNS.xxxxxx

[69] Jdid, M., Alhabib, R., Khalid, H. E., & Salama, A. A. (2022). The Neutrosophic treatment of the static model for the inventory management with safety reserve. *International journal of Neutrosophic science (IJNS), 18*(2), 262-271. https://doi.org/10.54216/IJNS.180209

[70] Mondal, B., Garai, A., Mukhopadhyay, A., & Majumder, S. K. (2021). Inventory policies for seasonal items with logistic-growth demand rate under fully permissible delay in payment: A Neutrosophic optimization approach. *Soft computing, 25*(5), 3725–3750. https://doi.org/10.1007/s00500-020-05402-9

[71] Haq, A., Gupta, S., & Ahmed, A. (2021). A multi-criteria fuzzy Neutrosophic decision-making model for solving the supply chain network problem. *Neutrosophic sets and systems, 46*, 50-66. https://fs.unm.edu/NSS/MulticriteriaFuzzyNeutrosophic5.pdf

[72] Piper, R. G. (1982). *Fish hatchery management*. US department of the interior, fish and wildlife service. https://dn790002.ca.archive.org/0/items/fishhatcherymana00pipe/fishhatcherymana00pipe.pdf

[73] Kar, C., Mondal, B., & Roy, T. K. (2018). An inventory model under space constraint in Neutrosophic environment: A Neutrosophic geometric programming approach. *Neutrosophic sets and systems, 2*, 93 – 109. https://doi.org/10.5281/zenodo.1408663

[74] Islam, S. (2021). Fuzzy multi-objective inventory model for deteriorating items, with shortages under space constraint: Neutrosophic hesitant fuzzy programming approach. In *Neutrosophic operational research: Methods and applications* (pp. 197-219). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-030-57197-9_11

[75] Ahmad, F., Adhami, A. Y., & Smarandache, F. (2019). Neutrosophic optimization model and computational algorithm for optimal shale gas water management under uncertainty. *Symmetry*, *11*(4), 544. https://doi.org/10.3390/sym11040544

[76] Islam, S., & Kundu, T. (2018). Application of Neutrosophic optimization technique on multi-objective reliability optimization model. *Neutrosophic sets and systems, 2, 13-27.* https://www.researchgate.net/publication/325106961

[77] Moorthy, C. B., Rajani, D., Pushpalatha, A. P., Ramya, S., Selvaraj, A., & Tiwari, M. (2024). Enhancing inventory management through advanced technologies and mathematical methods: Utilizing Neutrosophic fuzzy logic. *International journal of Neutrosophic science (IJNS)*, *24*(4). https://www.researchgate.net/profile/Mohit-Tiwari-6/publication/381095530_

[78] Jdid, M., Alhabib, R., & Salama, A. A. (2023). The static model of inventory management without a deficit with Neutrosophic logic. *Research on the topics of Neutrosophic operations research*, *16*(1), 24-48. https://doi.org/10.54216/IJNS.160104