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A Comparative Study of Continued Fraction and AI Methods for Polynomial Pell Equations

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Abstract

In this paper, we provide an overview of the comparative analysis of solving the Pell's equation $x^2-Dy^2=1$ and the corresponding Pell's type equation $x^2-Dy^2=-1$, where D is a given quadratic polynomial that is not a perfect square, by using tools based on generative artificial intelligence. The use of artificial intelligence today has a large and diverse application in society and science. It is important to carefully consider when, to what extent and whether to use artificial intelligence tools in some learning, research or analysis. Therefore, it is very meaningful to test AI tools in its wide range of applications. The use of artificial intelligence in mathematics is not a new form of research, but more modern and technologically improved with the respect to the previously used tools. In this paper, we compared the success of using AI in solving equations $x^2-dy^2=1$ and $x^2-dy^2=-1$. The analysis clearly indicates that both tools still show a high degree of vagueness, a poor strategy and a uniform approach in the selection of methods for solving equations. Since ChatGPT and Gemini are one of the most available artificial intelligence tools a comparison of the solutions provided by these tools is conducted. In addition to the given artificial intelligence solutions, we also present classical mathematical solutions. The number of correct, incorrect and undetermined answers is used as a measurement for the success these tools in solving the mentioned equations.

Keywords: Pell's equation, equations of Pell's type, generative AI, ChatGPT, Gemini.

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1|Introduction

An algebraic equation of the form

$$x^2 - Dy^2 = C \tag{1}$$

with two variables x and y, where D is a natural number and C is an integer, is called a Pell's type equation. For $C = 1$, we get an algebraic equation (Diophantine equation) of the form

$$x^2 - Dy^2 = 1, \tag{2}$$

which is called Pell's equation.

When $D = m^2$, where m is a natural number, i.e. when D is a perfect square, Pell's equation (1) has the form

$$x^2 - (my)^2 = (x - my)(x + my) = 1, \tag{3}$$

from which it follows

$$x - my = x + my = \pm 1. \tag{4}$$

Based on equality (4) we obtain trivial solutions of the given equation, i.e. $(x, y) = (\pm 1, 0)$. Therefore, due to the triviality of the solution, the case of D as a perfect square will not be considered. In this paper, we exclusively consider the cases when D is not a perfect square [2], [3], [4] and [7].

The study of this form of algebraic equations has a long and rich mathematical history. This type of equation was among the first to be dealt with by Archimedes (287 BC - 212 BC), studying the livestock problem and setting it up gives the form of the equation. Besides him, Diophantus (200 BC - 284 BC) also dealt with this problem [2].

Indian mathematicians Brahmagupta [2] in the sixth century, and Jayadeva and Bhaskara II in the 11th century, gave a special contribution to the methodical study of Pell's type algebraic equations. Brahmagupta gave a method for finding infinitely many solutions of a Pell's type equation based on one known particular solution, while Jayadeva and Bhaskara II gave an algorithm for finding the first solution. Although the algebraic equation (2) was named after the English mathematician John Pell (1611-1685), its study and development cannot be directly linked to Pell. The name is thought to have originated from a misattribution of credit to Pell by Leonhard Euler (1707-1783). In the 17th and 18th centuries, Pierre de Fermat (1601-1665), Leonhard Euler and Joseph Louis Lagrange (1736-1813) stand out for finding methods for solving these equations, with Lagrange being the first to provide a rigorous proof of the proposed method. In this paper, we will focus on the analysis and method of solving Pell's equation (2) and Pell's type equation

$$x^2 - Dy^2 = -1 \tag{5}$$

using the continued fraction method.

2|Mathematical Background and the Continued Fraction Method for Solving Pell'S Equations.

In this chapter, we will provide an overview of the basic mathematical theory of continued fractions and a description of the continued fraction method for solving Pell's equation (2) and Pell's type equation (5).

2.1|Continued Fraction

Let α be a real number and let a_0 be defined by $a_0 := \lfloor \alpha \rfloor$. If $a_0 \neq \alpha$ let $\alpha = a_0 + \frac{1}{\alpha_1}$, then let $a_1 = \lfloor \alpha_1 \rfloor$, for $\alpha_1 > 1$. For $\alpha_1 \neq a_1$, then $\alpha_1 = a_1 + \frac{1}{\alpha_2}$, where $\alpha_2 > 1$ and further $a_2 = \lfloor \alpha_2 \rfloor$. We can keep repeating this process until we get that $\alpha_n = a_n, n \in \mathbb{N}$. When $\alpha_n = a_n, n \in \mathbb{N}$ then the relation (6) holds [3], [7].

$$[a_0, a_1, a_2, ..., a_n] = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\ddots + \cfrac{1}{a_{n-1} + \cfrac{1}{a_n}}}}}, \tag{6}$$

The relation (6) represents a finite continued fraction written in a simpler form $[a_0, a_1, a_2, ..., a_n]$. If $\alpha \neq a_n$ for each $n \in \mathbb{N}$ the relation (6) becomes

$$[a_0, a_1, a_2, \ldots, a_n, \ldots] = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\ddots + \cfrac{1}{a_{n-1} + \cfrac{1}{a_n + \cfrac{1}{\ddots}}}}}}, \tag{7}$$

Expension into a simple continued fraction is finite in the case when α is a rational number, and its expansion into a continued fraction can also be obtained using the Euclidean algorithm, where $a_0, a_1, a_2, \ldots$ represent the quotients from the Euclidean algorithm applied to the numerator and denominator of a given rational number. Every rational number

$$\frac{p_k}{q_k} = [a_0, a_1, a_2, \ldots, a_k] \tag{8}$$

for $k \leq n$ is called the k-th convergent of $[a_0, a_1, a_2, \ldots, a_n]$. The numerator p_k and the denominator q_k, the k convergent expansion into a continued fraction of $[a_0, a_1, a_2, \ldots, a_n]$ satisfy the following recursive relations

$$p_k = a_k p_{k-1} + p_{k-2}, \quad p_0 = a_0, \quad p_1 = a_0 a_1 + 1, \tag{9}$$
$$q_k = a_k q_{k-1} + q_{k-2}, \quad q_0 = 1, \quad q_1 = a_1. \tag{10}$$

for $k \in \{2, 3, 4, \ldots, n\}$. In the case when α is an irrational number, the expansion into a continued fraction is infinite, i.e. $[a_0, a_1, a_2, \ldots, a_n, \ldots]$, and the values $a_0, a_1, a_2, \ldots, a_n$ in the relations (6) and (7) are called partial quotients. In this paper, we will specifically consider the case of periodic infinite continued fractions. A continued fraction $[a_0, a_1, a_2, \ldots, a_n, \ldots]$ is periodic if there are $l \in \mathbb{N}$ and $m \in \mathbb{N}$ such that $a_k = a_{k+l}$, for each $k \geq m$. Then we label l as the length of the period of the continued fraction with notation

$$\alpha = [a_0, a_1, a_2, \ldots, a_{m-1}, \overline{a_m, a_{m+1}, \ldots, a_{m+l-1}}], \tag{11}$$

where $\overline{a_m, a_{m+1}, \ldots, a_{m+l-1}}$ means that the numbers a_m, a_{m+1}, …, a_{m+l-1} are the one that are repeating. For $m = 0$, we will get a purely periodic continued fraction, i.e. $[\overline{a_0, a_1, a_2, \ldots, a_{l-1}}]$, [3], [7].

By quadratic irrationality we mean every irrational number α of the form $a + b\sqrt{d}$, where $a, b \in \mathbb{Q}$ and $d \in \mathbb{N}, d \neq \square$ that satisfies the given quadratic equation with rational coefficients. If α is a quadratic irrationality then the expansion into a continued fraction is periodic and vice versa. The proof of this statement was given by Euler and Lagrange [7].

For a given quadratic irrationality $\alpha = \frac{s_0+\sqrt{d}}{t_0}$, where $s_0, t_0 \in \mathbb{Z}$, $t_0 \neq 0$, $t_0|(d - s_0^2)$, $d \in \mathbb{N}$ and $d \in \mathbb{N}$. For $i > 0$, the partial quotients are obtained as follows:

$$a_i = \lfloor \alpha_i \rfloor, \quad s_{i+1} = a_i t_i - s_i \quad t_{i+1} = \frac{d - s_{i+1}^2}{t_i} \quad \alpha_{i+1} = \frac{s_{i+1} + \sqrt{d}}{t_{i+1}} \tag{12}$$

We will use this algorithm in expanding into a continued fraction while solving Pell's equation (2) and Pell's type equation (5).

2.2|The Continued Fraction Method for Solving Pell'S Equations

There are a number of different methods for solving Pell and Pell's type equations: continued fraction method, recursive method, Euler method, unimodular plane transformation method. One of the methods is the continued fraction method, which we will consider in this paper mathematically and through the application of artificial intelligence. The first step in solving algebraic equations of the form (2) and (5) is to find the expansion of the number or expression into a simple continued fraction as it is described in the previous section. Due to the symmetry of Pell's equation, it is sufficient to observe the solutions only in the set of natural numbers, i.e. if the ordered pair $(x, y), x, y \in \mathbb{N}$ is a solution of the equation (2), then its solutions are also $(x, -y)$, $(-x, y)$ and $(-x, -y)$. It is obvious that one solution is $(1, 0)$ and due to its triviality we will not consider it. By the fundamental solution (x_1, y_1) we mean a non-trivial solution where x_1 and y_1 are the smallest natural numbers such that the equation (2) is satisfied, and we write it as follows

$$x_1 + y_1\sqrt{D}. \tag{13}$$

Based on the fundamental solution (13) of the equation (2) we determine all other solutions in the set of natural numbers using the relation

$$x_n + y_n\sqrt{D} = (x_1 + y_1\sqrt{D})^n \in \mathbb{N}.$$

As $n \in \mathbb{N}$ it is obvious that the equation (2) has infinitely many solutions. The given relation is stated as Theorem 10.11 in [3], where its proof can be found. Based on Theorem 10.12 in [3] the value of the trivial solution becomes obvious when finding all natural solutions of Pell's equation (2) using relations

$$x_{n+2} = 2x_1x_{n+1} - x_n, \tag{14}$$
$$y_{n+2} = 2x_1y_{n+1} - y_n, n \geq 0. \tag{15}$$

and using the fundamental solution. In this way, a series of solutions is obtained in ascending order [3], [7].

It is obvious that the role of the fundamental solution in finding all solutions is significant. The key moment in solving Pell's equation (2) is to find the fundamental solution. The question arises how to determine the fundamental solution. One of the most effective ways is to establish a connection between the solution of Pell's equation (2) and the Diophantine approximation of the number or expression $\sqrt{D}$, where D is not a perfect square.

The solutions of Pell's and the Pell's type equation depend on the length of the period l of the expansion of the number or expression $\sqrt{D}$ into a continued fraction (Theorem 10.20 in [3]). Each Pell's equation (2) is also given with a relation

$$(x, y) = (p_{nl-1}, q_{nl-1}), n \in \mathbb{N}, \tag{16}$$

if the length of the period l is an even number where the fundamental solution is (p_{l-1}, q_{l-1}) i.e. or with the relation

$$(x, y) = (p_{2nl-1}, q_{2nl-1}), n \in \mathbb{N}, \tag{17}$$

if the length of the period l is an odd number, where fundamental solution of the equation (2) (p_{2l-1}, q_{2l-1}). Unlike Pell's equation, each Pell's type equation does not always have a solution. In the case when the length of the period l is an even number, the Pell equation (5) will not have a solution, and in the case when the length of the period l is an odd number, the solutions are given by

$$(x, y) = (p_{(2n-1)l-1}, q_{(2n-1)l-1}). \tag{18}$$

Especially the Pell's type equation (5) will not have a solution in the case where $D \equiv 3 \bmod 4$, in [3], [7].

3|Discussion and Analysis of the Results

In this paper, we investigated how AI, specifically using ChatGPT and Gemini, solves and investigates polynomial Pell equations. For this research, we chose seven carefully selected polynomial expressions for D, which we used to form the Pell equations. The selection was not random but carefully and precisely chosen. The examples were chosen based on the working principles of generative AI. To investigate the operation of methods and modes of operation, we have chosen the following polynomial Pell equations.

$$x^2 - ((89t-44)^2 + 110t - 54)y^2 = 1, \quad (19)$$

$$x^2 - ((70t+1)^2 + 58t + 1)y^2 = 1, \quad (20)$$

$$x^2 - ((169t+1)^2 + 140t + 1)y^2 = 1, \quad (21)$$

$$x^2 - ((29t+1)^2 + 58t + 1)y^2 = 1, \quad (22)$$

$$x^2 - (a^2-4)y^2 = 1, a > 3, a \text{ odd}, \quad (23)$$

$$x^2 - (a^2+4)y^2 = 1, a > 1, a \text{ odd}, \quad (24)$$

$$x^2 - ((na)^2 - a)y^2 = 1, n \in \mathbb{N}, a > 1. \quad (25)$$

The basic Pell's equation using the method previously described in section 2 immediately offers a solution to equation (4) whose analysis and method of solution are also given in the discussion of this paper for selected problems.

Solving equations of the form $x^2 - Dy^2 = \pm 1$ is formally carried out entirely algorithmically. Solving the mentioned equations is carried out in several clearly defined and determined steps, which we described in the previous section. Below, we provide an overview of the mathematical solution, and AI tested on ChatGPT and Gemini. The first step of solving $x^2 - Dy^2 = \pm 1$ is the determination of the development into a simple continued fraction of the number $\sqrt{D}$. In Table 1, we provide an overview of the mathematical solutions [7] and ChatGPT.

Table 1. Review the math solution and the solution given by ChatGPT for developing $\sqrt{D}$ into a simple continued fraction.

$\sqrt{D}$	Mathematical solution	ChatGPT solution
$\sqrt{(89t-44)^2+110t-54}$	$[89t-44; \overline{1,1,1,1,1,1,1,1,1,1,178t-88}]$	$[89t-44; \overline{1,1,1,1,1,1,1,1,1,1,178t-88}]$
$\sqrt{(70t+1)^2+58t+1}$	$[70t+1; \overline{2,2,2,2,2,140t+2}]$	$[70t+1; \overline{2,2,2,2,2,140t+2}]$
$\sqrt{(169t+1)^2+140t+1}$	$[169t+1; \overline{2,2,2,2,2,2,338t+2}]$	$[169t+1; \overline{2,2,2,2,2,2,338t+2}]$
$\sqrt{(29t+1)^2+58t+1}$	$[29t+1; \overline{2,2,2,2,58t+2}]$	$[29t+1; \overline{1,29,1,58t+2}]$
$\sqrt{a^2-4}, a>3$ for a odd	$[a-1; \overline{1,\frac{a-3}{2},2,\frac{a-3}{2},1,2a-2}]$	$[a-1; \overline{1,\frac{a-3}{2},2,\frac{a-3}{2},1,2a-2}]$
$\sqrt{a^2+4}, a>1$ for a odd	$[a; \overline{\frac{a-1}{2},1,1,\frac{a-1}{2},2a}]$	$[a; \overline{\frac{a-1}{2},1,1,\frac{a-1}{2},2a}]$
$\sqrt{(na)^2-a}, n\in\mathbb{N}, a>1$	$[na-1; \overline{1,2n-2,1,2(na-1)}]$	$[na-1; \overline{1,2n-2,1,2(na-1)}]$

ChatGPT as a generative AI tool that works on NLP algorithms for example (19) gave the correct development into a simple continued fraction, but only for $t = 1$, without performing the general form of the development of expression $\sqrt{(8t-24)^2 + 110t - 54}$ into a simple continued fraction. In example (23) he did not perform the correct form, while in the other cases he provided the requested solutions. We performed the same analysis using the AI Gemini tool and obtained the results, which are presented in the following table.

Table 2. Review the math solution and the solution given by Gemini for developing $\sqrt{D}$ into a simple continued fraction.

$\sqrt{D}$	Mathematical solution	Gemini solution
$\sqrt{(89t-44)^2+110t-54}$	$[89t-44; \overline{1,1,1,1,1,1,1,1,1,1,178t-88}]$	It is not determined.
$\sqrt{(70t+1)^2+58t+1}$	$[70t+1; \overline{2,2,2,2,2,140t+2}]$	$10\sqrt{51} = [71; \overline{2,2,1,1,2,1,142}]$
$\sqrt{(169t+1)^2+140t+1}$	$[169t+1; \overline{2,2,2,2,2,2,338t+2}]$	$[170; \overline{2,2,2,2,\ldots}]$
$\sqrt{(29t+1)^2+58t+1}$	$[29t+1; \overline{2,2,2,2,58t+2}]$	$[29t+1; \overline{1,29,1,58t+2}]$
$\sqrt{a^2-4}, a>3$ for a odd	$[a-1; \overline{1,\frac{a-3}{2},2,\frac{a-3}{2},1,2a-2}]$	$[a-1; \overline{1,\frac{a-3}{2},2,\frac{a-3}{2},1,2a-2}]$
$\sqrt{a^2+4}, a>1$ for a odd	$[a; \overline{\frac{a-1}{2},1,1,\frac{a-1}{2},2a}]$	It is not determined.
$\sqrt{(na)^2-a}, n\in\mathbb{N}, a>1$	$[na-1; \overline{1,2n-2,1,2(na-1)}]$	$[na-1; \overline{1,2n-2,1,2(na-1)}]$

Based on the given mathematical solutions and solutions obtained with AI tools, the length of the development period was determined as a simple continued fraction. An overview of the results is given in the Table 3.

TABLE 3. Overview of obtained periods l in the development of the continued fraction in the mathematical, ChatGPT and Gemini solution

$\sqrt{D}$	Mathematical solution	ChatGPT	Gemini
$\sqrt{(89t-44)^2+110t-54}$	$l=11$	$l=4$	It is not determined.
$\sqrt{(70t+1)^2+58t+1}$	$l=6$	$l=6$	$l=7$
$\sqrt{(169t+1)^2+140t+1}$	$l=7$	$l=7$	$l=\infty$
$\sqrt{(29t+1)^2+58t+1}$	$l=5$	$l=4$	$l=4$
$\sqrt{a^2-4}, a>3$ for a odd	$l=6$	$l=6$	$l=6$
$\sqrt{a^2+4}, a>1$ for a odd	$l=5$	$l=5$	It is not determined.
$\sqrt{(na)^2-a}, n\in\mathbb{N}, a>1$	$l=4$	$l=4$	$l=4$

In the development into a simple continued fraction, the development period allows us to calculate the convergences and determine the fundamental solution of the equation $x^2-Dy^2=\pm1$. For example 1, ChatGPT provided only the development for $t=1$ and therefore only enabled the solution of equation $x^2-312y^2=\pm1$, but not the general solution of the required equation. In determining the period, Gemini did not determine the period in examples 1 and 6 at all, while for examples 2 and 3 it gives the development and period only for specific t, which again does not enable the general solution of the required equations. The analysis of the success of determining the development into chain fractions of ChatGPt and Gemini is given in the following table.

TABLE 4. Success in solving the expansion of the number $\sqrt{D}$ into a simple continued fraction

Tool	True %	False %	It is not determined. %
ChatGPT	72	14	14
Gemini	29	14	57

If we observe the success of determining the expansion of the number $\sqrt{D}$ into a simple continued fraction by artificial intelligence, we will see that ChatGPT had a success rate of 72%, while the success rate of Gemini is 29%. ChatGPT and Gemini had the same failure rate (incorrect solutions) of 14%. Artificial intelligences' tools show the greatest divergence in cases when they fail to determine any concrete solution. ChatGPT had 14% of such cases and Gemini 57%.

The second step in solving the Pell's equation and Pell's type equations is determining the fundamental solutions. We determine the fundamental solution of the corresponding Pell's equation based on given claims and theoretical foundations that we gave in section 2. In the following Table we give an overview of the fundamental solutions obtained by classical calculation, i.e. solving equations by mathematicians.

TABLE 5. The fundamental solution of the Pell's equations solved by mathematician.

Example	Mathematical solution
$x^2-((89t-44)^2+110t-54)y^2=1$	$(x_1,y_1)=(77546590t^2-75598291t+18424736, 871310t-424709)$
$x^2-((70t+1)^2+58t+1)y^2=1$	$(x_1,y_1)=(4900t+99,70)$
$x^2-((169t+1)^2+140t+1)y^2=1$	$(x_1,y_1)=(675753260t^2+11309649t+47321, 3998540t+33461)$
$x^2-((29t+1)^2+58t+1)y^2=1$	$(x_1,y_1)=(243890t^2+24563t+618, 8410t+437)$
$x^2-(a^2-4)y^2=1$, $a>3$, a odd	$(x_1,y_1)=(\frac{a^3-3a}{2},\frac{a^2-1}{2})$
$x^2-(a^2+4)y^2=1$, $a>1$, a odd	$(x_1,y_1)=(\frac{a^6+6a^4+9a^2+2}{2},\frac{a^5+4a^3+3a}{2})$
$x^2-((na)^2-a)y^2=1$, $n\in\mathbb{N}, a>1$	$(x_1,y_1)=(2n^2a-1,2n)$

In the following tables, we will present the solution of selected problems by artificial intelligence. In Table 6. we present the solutions obtained using the ChatGPT tool.

We can see that in three cases the artificial intelligence tool ChatGPT gave only a trivial solution $(x_0,y_0)=(1,0)$, while in three cases it gives solutions, and in one case it did not offer a concrete solution to the problem at all. The table 7 shows the results of selected problems given by artificial intelligence tool Gemini.

TABLE 6. The fundamental solution of the Pell's equations solved by ChatGPT.

Example	ChatGPT
$x^2-((89t-44)^2+110t-54)y^2=1$	Trivial solution $(x_0,y_0)=(1,0)$
$x^2-((70t+1)^2+58t+1)y^2=1$	Trivial solution $(x_0,y_0)=(1,0)$
$x^2-((169t+1)^2+140t+1)y^2=1$	Trivial solution $(x_0,y_0)=(1,0)$
$x^2-((29t+1)^2+58t+1)y^2=1$	$(x_1,y_1)=((29t+2)^2-1,29t+2)$
$x^2-(a^2-4)y^2=1$, $a>3$, a odd	$(x_1,y_1)=\left(\frac{a^3-3a}{2},\ \frac{a^2-1}{2}\right)$
$x^2-(a^2+4)y^2=1$, $a>1$, a odd	Isn't detrmined.
$x^2-((na)^2-a)y^2=1$, $n\in\mathbb{N},a>1$	$(x_1,y_1)=(2n^2a+1,2n)$

TABLE 7. The fundamental solution of the Pell's equations solved by Gemini

Example	Gemini
$x^2-((89t-44)^2+110t-54)y^2=1$	Isn't determined.
$x^2-((70t+1)^2+58t+1)y^2=1$	$(4900t+99,70)$
$x^2-((169t+1)^2+140t+1)y^2=1$	Isn't determined.
$x^2-((29t+1)^2+58t+1)y^2=1$	Isn't determined.
$x^2-(a^2-4)y^2=1$, $a>3$, a odd	$(x_1,y_1)=(\frac{a(a^2-3)}{2},\frac{a^2-1}{2})$
$x^2-(a^2+4)y^2=1$, $a>1$, a odd	$(\frac{a^2}{2}+1,\frac{a}{2})$
$x^2-((na)^2-a)y^2=1$, $n\in\mathbb{N},a>1$	$(2n^2a+1,2n)$

It is obvious that Gemini could not detect any specific solution in three cases, while it offers its own solutions for four problems. Based on obtained results, we can conclude that ChatGPT gave 29% correct solutions excluding trivial solutions, 14% incorrect and that in 14% cases it failed to determine any solutions. Gemini as an artificial intelligence tool gives 57% correct, no incorrect answers, and 43% indeterminate solutions.

TABLE 8. The fundamental solution of the Pell's equations solved by mathematician, ChatGPT and Gemini.

Example	Mathematical solution	ChatGPT	Gemini
$x^2-((89t-44)^2+110t-54)y^2=-1$	$(x_1,y_1)=(7921t-3861,89)$	Isn't dertermined.	Isn't determined.
$x^2-((70t+1)^2+58t+1)y^2=-1$	$(x_1,y_1)=(4900t+99,70)$	Isn't dertermined.	Isn't determined.
$x^2-((169t+1)^2+140t+1)y^2=-1$	$(x_1,y_1)=(28561t+239,169)$	Isn't dertermined.	Isn't determined.
$x^2-((29t+1)^2+58t+1)y^2=-1$	$(x_1,y_1)=(841t+41,29)$	Isn't dertermined.	Isn't determined.
$x^2-(a^2-4)y^2=-1$, $a>3$, a odd	Solution does not exist.	$(x_1,y_1)=(\frac{a^3-3a}{2},\frac{a^2-1}{2})$	$(32a+1,8)$
$x^2-(a^2+4)y^2=-1$, $a>1$, a odd	$(x_1,y_1)=(\frac{a^3+3a}{2},\frac{a^2+1}{2})$	Isn't dertermined.	$(\frac{a^3+3a}{2},\frac{a^2+1}{2})$
$x^2-((na)^2-a)y^2=-1$, $n\in\mathbb{N},a>1$	Solution does not exist.	$(2n^2a+1,2n)$	Isn't determined.

Mathematicians use convergent calculations in order to obtain the fundamental solution. We noticed that most mathematicians calculate convergents algorithmically, but not ChatGPT. Instead of using the method of calculating convergents, ChatGPT uses the method for solving Diophantine equations based on the divisibility of numbers. Although the algorithm immediately offers a solution to Pell's equation (5), artificial intelligence tools do not immediately offer a solution to this equation. Even when it is known (based on properties of the equation) that the equation has no solution, these tools will not provide information that there is no solution. A comparative analysis of fundamental solutions Pell's problem (5) obtained by mathematicians, ChatGPt and Gemini is presented in Table 8.

4|Conclusion

Every artificial intelligence tool can be suitable for the user who is experienced in recognizing illogicalities and solutions' inconsistencies and answers to the problems or questions that arise [8]. There are many different aspects from which can be approached in studing the comparison of ChatGPT and Gemini and their working styles. The authors are based on different approaches to generative artificial intelligence in education, but one of the interesting overviews of advantages and disadvantages is provided in Orange's paper [6].

Our research shows that ChatGPT and Gemini have limited abilities to solve Pell's equation as well as Pell's type equation $x^2 - dy^2 = -1$. In addition to the limitations, we also point out the lack precision and reliability in the offered solutions. Therefore, the user should carefully and critically approach his conclusions because they are not always correct. In other words, ChatGPT and Gemini chose the same strategy in solving Pell's equation as well in solving Pell's type equation. The chosen method was based on the divisibility of numbers. According by chosed strategy, it is clearly shown that artificial intelligence tools have a uniform, but also uncritical approach to problem solving. Here it is particularly interesting to point out that these artificial intelligence tools solved the given problems by reducing to the mere application of algorithms and procedures, thus the results obtained are even more interesting. Beside mentioned disadventages, one of the important advantages is an accurate overview of the theoretical basis that is provided for each problem. But it becomes disadvantage when it comes to the implementation. Implemented method is not consistent with the described overview.

According to some authors, ChatGPT gives better results in creative solutions and parts of problems, while they point out that Gemini gives better results in scientific and technical applications, [1] and [5]. Based on the results obtained in this research, it can not be concluded that there is a significant difference between the use of artificial intelligence tools in solving Pell'e eqaution $x^2 - Dy^2 = 1$ and Pell's type equation $x^2 - Dy^2 = -1$.

Author Contribution

All authors have read and agreed to the published version of the manuscript.

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